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OF
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Ordinary Meeting, 9th May, 1917.

Held at Burlington House, London.

F. J. CHESHIRE, President, in the Chair.

The Minutes of the previous Ordinary Meeting were read, confirmed, and signed.

The following were balloted for and duly elected to the Society :—

As Members

Robert O. Burns, A.R.T.C., Glasgow.
Harry Arthur Carter, Messrs. J. H. Dallmeyer, Ltd., London.
James Gray Docherty, B.Sc., Engineer, Glasgow.
Charles Fabry, Professor of Physics at the University of Marseilles, Paris.
Robert C. Fisher, Messrs. Heath and Co., Ltd., London.
Andrew Robson, B.Sc., A.M.I.Mech.E., Glasgow.

It was announced that the undermentioned candidates had been approved by the Council for the ballot. The name of the proposer is appended :—

For the Class of "Member."

Frank A. Amsden, London (Wm. Shackleton).
F. F. S. Bryson, M.A., B.Sc., London (F. Cheshire).
Charles E. Heath, F.R.M.S., London (M. Blood).

The following paper by Mr. P. F. Everitt was discussed, after which a short address was given by Mr. J. S. Dow, on "The Eye in Relation to some Practical Lighting Problems."

On the Design and Testing of Telescope Objectives.

By P. F. EVERITT, B.Sc.

With contributions by T. SMITH, B.A., and S. D. CHALMERS, M.A.

Introduction.

THE function of the objective or object-glass of a telescope is to form as perfect an image as possible of an object area which is usually at a great distance and covers an angle of view rarely exceeding 5° from the optical axis.

A simple positive lens was first used for this purpose, but the performance of a telescope with such an objective was poor and its usefulness restricted by the aberrations of the simple lens; in course of time improvements were effected by the construction of compound objectives, and it is the design and testing of these compound objectives that I propose to discuss in this paper.

The Four Principal Aberrations of the Telescope Objective.

In any lens system when the rays under consideration are restricted to extremely narrow pencils indefinitely close to the axis, Gauss showed that the performance of the system depended on the properties of six points on the axis, the two focal points, the two principal points, and the two nodal points, which, when the refractive indices of the first and last media are the same, coincide with the principal points. As in practically every case, a telescope is so constructed that the media in front of and behind the objective are the same, viz., air, the six points mentioned are reduced to four, the two focal points and the two principal points. The properties of these fundamental points are well known to all students of geometrical optics, and I intend only to deal with such as are necessary for our purpose.

Fig. 1 shows the optical axis of a lens system; F_1, F_2 are the two focal points, H_1, H_2 the principal points, and the two planes drawn through H_1, H_2 perpendicular to the axis are known as the principal planes. The distances H_1F_1, H_2F_2 are equal, and are known as the anterior and posterior focal lengths, or more shortly, the focal lengths. We have further the well-known geometrical construction that a ray parallel to the axis before refraction behaves as if it were produced to the first principal plane, proceeds from there to the second principal plane in a direction parallel to the axis, and is then deviated to pass through F_2, where the image of the infinitely distant object on the axis is formed.

In actual lens systems as contrasted with the ideal system, the four points F_1, H_1, H_2, F_2 are not fixed but subject to changes depending on the particular rays considered; in other words, the system is subject to aberrations.

Variation of the position of F_2 with the wave-length (or colour) of the light forming the pencil is defined as " chromatic aberration of the intersection-distances "* and variation of the length H_2F_2 with the wave-length of the light is called " chromatic aberration of the focal lengths." These two terms will be referred to more shortly as chromatic aberration and the chromatic difference of magnification.

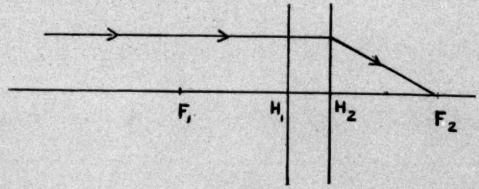

Fig. 1.

The Gauss theory of the geometrical representation of the formation of images illustrated in Fig. 1 involved the condition that the light pencils were extremely narrow and indefinitly close to the axis; the lens system was supposed to be one of indefinitely small aperture, and consequently would only transmit an indefinitely small amount of light. An actual lens system to be of use must form a bright image, *i.e.*, transmit a considerable amount of light, and we must now consider the effect produced by enlarging the aperture of the system.

Consider a narrow pencil of monochromatic light coming from an infinitely distant object on the axis and incident on the system at a considerable distance from the axis. It will be parallel to the axis and will intersect it after refraction at a point near F_2, if it does not actually pass through F_2. The variation of this point from the position of F_2 for the corresponding pencil close to the axis is called the longitudinal spherical aberration of the pencil.

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* The " intersection-distance " is the distance of F_2 from the vertex of the posterior surface of the lens system.

The spherical aberration will vary with the distance of the incident pencil from the axis, and is a direct consequence of the law of refraction being a relation between the sines of the angles of incidence and refraction, and not the angles themselves.

The whole aperture of the lens system may be regarded as filled with an infinite number of these narrow pencils, which will intersect the axis at various points, giving a circular patch of light (if the aperture be circular), which is called the circle of least confusion. When the spherical aberration of a system is mentioned without defining the distance from the axis of the particular narrow pencil involved it will be understood that the pencil is incident at the edge of the aperture, and is consequently at the maximum distance from the axis.

If in a lens system the spherical aberration just defined be eliminated, the monochromatic image of an object point on the axis will be another point on the axis. In order that the image of a small area or element of surface on the axis and perpendicular to it shall be reproduced as a corresponding area in the image when the whole aperture of lens system is used, it can be shown that the condition to be fulfilled is that $\frac{\sin u}{\sin u^1}$ shall be constant where u and u^1 are the angles with the axis formed by any narrow pencil before and after refraction in proceeding from the object to the image, as shown in Fig. 2.

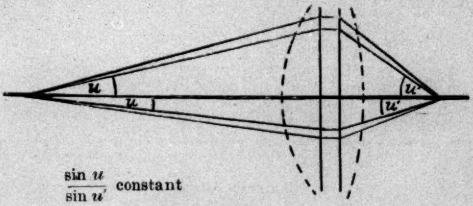

Fig. 2.

For an object at an infinite distance the angle u becomes infinitely small, and the condition reduces to another form. In its new form the condition becomes that the distance measured along the refracted ray from the focus F_2 to its intersection with the unrefracted ray produced shall be constant and equal to the focal length H_2F_2; in other words, the points of intersection of the unrefracted ray and refracted rays shall lie on a circle, centre F_2 and radius H_2F_2. The distances $A F_2$, $B F_2$.............$F F_2$ in Fig. 3 are equal to H_2F_2, the focal length of the axial pencil, and are in reality the focal lengths of the individual narrow pencils forming the complete cone of light transmitted by the lens system.

Since the magnification varies as the focal length, the sine condition expressed in its simplest form is that the magnification due to the lens system shall be constant over the whole aperture.

The four principal aberrations of the telescope objective have now been described, and it will be noticed that two of them, chromatic and spherical aberration, are aberrations of the intersectional distances; while the other two, chromatic difference of magnification and the error against the sine condition, are aberrations of the focal lengths.

Other Aberrations.

Lens systems in general are subject to other aberrations, such as astigmatism of oblique pencils, curvature of field, distortion, chromatic aberrations of higher orders, &c., but as will be seen later, there will not be sufficient constructional elements available to eliminate these; and, further, they are never present to such a degree as to injuriously affect the performance of telescope objectives, the field of view being so small that they may be neglected, except in the case of objectives used in the production of photographic charts of the stars. In that case it is sufficient to determine the distortional errors introduced by the objectives, and allow for them in the reduction of the measurements of the negatives.

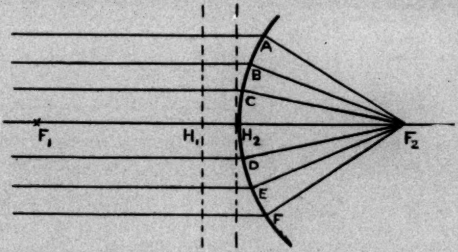

Fig. 3.

Residual Aberrations.

Consider an objective from which the four principal aberrations have been removed; it will satisfy the following conditions :—

(1) The axial pencil will be free from chromatic aberration for two wave-lengths, *e.g.*, the C and F lines of the spectrum.

(2) The axial pencil will be free from chromatic difference of the focal lengths for the same two wave-lengths.

(3) The axial and marginal pencils for any one wave-length, say C, will intersect the axis at the same point.

(4) The focal lengths of these two axial and marginal pencils will be equal.

Condition (1) only considers light of two particular wave-lengths, C and F, and gives no information as to what happens to light of any other wave-length that also passes through the system. In reality it is found that pencils of light of wave-lengths intermediate between C and F, and also for a little distance on either side of that interval come to a focus very close to the common focus of the C and F pencils, but do not coincide with it. There is thus a residual chromatic aberration which may or may not be sufficient to impair the efficiency of the system. If the aberration is removed for light of three different wave-lengths the amount of residual aberration is very much reduced. The residual chromatic aberration when light of two wave-lengths is brought to a common focus is frequently called " secondary spectrum," while the much smaller residual after bringing the foci of three wave-lengths to coincide is called " tertiary spectrum."

Condition (3) requires the axial and marginal pencils to intersect the axis at the same point for light of one wave-length C, and condition (1) also requires the axial pencil for F to pass through the same point; neither condition requires the marginal pencil for F to pass through this point, and in general it will not do so. The conditions laid down do not demand the spherical aberration to be corrected for more than one wave-length, and the residual spherical aberration for light of any other wave-length F is known as the chromatic difference of spherical aberration, and should not be confused with secondary spectrum.

Before passing on further it is important to note that condition (3) only refers to the axial and marginal pencils, and not to the intermediate pencils which also pass through the lens system and are found to intersect the axis at different points for the different concentric zones of the aperture of the system. These give residual spherical aberration which is frequently called zonal aberration, which, in the case when the spherical aberration for the marginal pencils is rigidly corrected, reaches a maximum for the zone distant from the axis $h/\sqrt{2}$, where h is the semi-diameter of the aperture.

Similar residual aberrations to those just mentioned with regard to conditions (1) and (3) also exist for conditions (2) and (4), but as they are of much smaller importance, they have not received the dignity of special names, nor need they be specially considered.

The Six Conditions in Order of General Importance.

Before passing on to the consideration of the possibilities offered by various types of lens systems it will be useful to state shortly the six conditions which it is important to fulfil as far as practicable in the actual work of designing telescope objectives. Although special circumstances will require special treatment, the conditions in order of their general importance are :—

(*a*) The objective shall have a given aperture and a given focal length.

(*b*) The spherical aberration shall be corrected for a specified wave-length.

(*c*) The chromatic aberration shall be corrected for two wave-lengths chosen with regard to the purpose for which the system is to be used.

(*d*) The sine condition shall be fulfilled for the wave-length specified in condition (*b*).

(*e*) The chromatic aberration shall be corrected for a third specified wave-length.

(*f*) The spherical aberration shall be corrected for a second specified wave-length (sometimes known as the Gauss condition).

The above six conditions will generally be sufficient, but attention should be paid to the chromatic difference of magnification, which will usually be negligible, although it is possible to construct a system in which this aberration would be so large as to seriously impair the performance of the objective.

Main Types of Telescope Objectives and the Possibilities they offer the Designer.

Cemented Doublets.

This type of objective is probably that in most general use to-day, and is universally used in the construction of prism binoculars. The constructional elements available are the three radii, choice of two glasses, and to an extremely limited extent the choice of two thicknesses.

The suitable choice of the three radii enables the designer to fulfil conditions (*a*), (*b*), and (*c*). Alteration of the two thicknesses within practical limits allows of a slight variation in the aberrations of the focal lengths, but the resulting change is so small that it is not usually appreciable. There still remains one more possibility; by a suitable choice of the pairs of glasses it is possible either to reduce the secondary spectrum or to satisfy the sine condition, and in the case of prismatic binocular objectives this latter condition should always be fulfilled.

H. Harting has investigated the problem very fully, and has published tables* by means of which it is possible to tell in a few minutes whether any given pair of glasses will satisfy not only conditions (*a*), (*b*), and (*d*), but also condition (*c*). If the pair of glasses is suitable, *i.e.*, will satisfy condition (*c*), the radii and thickness can be found from the tables, so that in about half-an-hour by means of these tables and a glass catalogue it is possible to obtain the approximate radii and thicknesses of the system.

Smith & Cheshire† have recently published tables, at the request of the Ministry of Munitions, showing the constructional data of cemented doublets for certain types of glasses and giving the approximate amount of spherical aberration and relative amount of coma remaining uncorrected.

Uncemented Doublets.

No telescope objective over 3 ins. in diameter, and also no objective where the utmost obtainable freedom from spherical aberration is required, as in collimator objectives, should be cemented on account of the strains set up owing to the difference in the co-efficients of expansion of the glasses.

In an uncemented doublet there are four radii available which will always allow of conditions (*a*), (*b*), (*c*), and (*d*) being satisfied without unduly restricting the choice of glass pairs. A suitable choice of glass pairs will then enable the designer either to reduce the secondary spectrum, as is frequently done, or to satisfy some other condition such as (*f*), although I am not aware of this latter condition having been fulfilled simultaneously with the sine condition (*d*) in any actual example of an objective.

Another possibility in this case is to choose a pair of glasses giving the flattest curves obtainable, thus reducing manufacturing costs, and also tending to reduce the residual zonal aberration.

Triplet Objectives.

Triplet objectives are of two types, being either doubly cemented or with one cemented contact and one air space.

The doubly cemented type of triplet offers the advantage of a smaller residual zonal aberration for the same pair of glasses and the same relative aperture than the doublets, and consequently is frequently employed when a relatively large aperture is required in conjunction with a comparatively short focal length. Owing to the designer having four radii available, it is possible to satisfy all four conditions (*a*), (*b*), (*c*), and (*d*), as in the case of an uncemented doublet, without the restrictions as to choice of glass pairs required to satisfy conditions (*d*) in the case of a cemented doublet.

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* Zeitschrift für Instrumentenkunde 1898, Heft 12; also Gleichen, Lehrbuch der Geometrischen Optik, p. 324.

† "Constructional Data of Small Telescope Objectives," by T. Smith, B.A., and R. W. Cheshire, B.A.

If three glass types are used instead of two, the chromatic corrections may be improved by uniting three wave-lengths instead of two, condition (*e*), but this will usually be at the expense of satisfying the sine condition (*d*).

It is easy to satisfy all the five conditions, (*a*) to (*e*), if the designer makes use of the second type of triplet having one cemented contact and one air space, thus obtaining another radius, which enables the fifth condition to be fulfilled.

Other Types of Objectives.

Other types of lens systems have been used as telescope objectives for special purposes ; they are really other types of instruments modified for the particular purpose for which they are required, and need not be further considered.

Choice of Type.

It is not intended to lay down any hard and fast rules as to choice of type, but the following short table may be of some use as a general guide :—

Ordinary glass pairs.	Cemented doublet	Small objectives generally.
	Uncemented doublet	Large objectives for astronomical use, collimators and general use where the utmost perfection of image is demanded, provided that ordinary achromatism suffices.
	Doubly cemented triplet	Small objectives where large aperture is required in conjunction with short focal length. The relative aperture may be pushed as far as $f/4$ or $f/6$ according to size.

Special choice of glass pairs to satisfy condition (*d*).	Cemented doublet ..	Small objectives generally, specially suitable for prismatic binoculars.
Special choice of glass pairs to improve the chromatic corrections.	Uncemented doublet..	Large objectives for astronomical use, collimators and general use where the utmost perfection of image is required together with the best possible chromatic corrections. Secondary spectrum eliminated. Relative apertures up to about *f*/18.
	Cemented triplet with one air space	Objectives where the uncemented doublet fails owing to the aperture being relatively small. The aperture of this type may be made as large as *f*/10.

Actual Calculation of the Objective.

Having decided upon the type of objective required and the particular glasses it is intended to use, the approximate radii are first obtained from tables* wherever possible, past experience in similar cases, or, in default of any other source, by calculation from the known formulæ† which lead to an approximate solution. This latter method is very rarely necessary, and so only the references are given.

The approximate radii and thicknesses are now known, and of course the aperture and focal length. In order to determine the actual aberrations, it is required to trace trigonometrically through the system, for each wave-length to be considered, a ray parallel to the axis and indefinitely close to it, called the axial ray, and another ray also parallel to the axis but incident on the objective at the extreme edge of its aperture, the edge ray.

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* (1) "Constructional Data of Small Telescope Objectives," T. Smith, B.A., and R. W. Cheshire, B.A. (2) "Additional Data for the Construction of Small Telescope Objectives," T. Smith, B.A., and R. W. Cheshire, B.A. (3) H. Harting, Zeitschrift für Instrumentenkunde, 1898, Heft 12; also Gleichen, Lehrbuch d. Geom. Optik, p. 323.

† Moser, Zeitschrift für Instrumentenkunde, 1887, Bd. 7, pp. 225 and 308; also Gleichen, Lehrbuch d. Geom. Optik, p. 315; H. Harting, Zeitschrift für Instrumentenkunde, 1900, Heft 8; also Gleichen, Lehrbuch d. Geom. Optik, p. 320.

The formulæ of L. von Seidel are very well adapted to this end, and are given below as far as they are required for the purpose of this paper. The notation used is very simple and if further details than those given are desired, reference should be made to Appendix III. of Professor Silvanus Thompson's translation of Lummer's "Photographic Optics."

The quantities used are :—

μ the index of refraction.

R the radius of the surface, positive when convex to the direction of the incident light.

A the distance from the vertex of the surface to the point of intersection with the axis of the refracted ray with regard to that surface, positive in the direction of the incident light.

H the height of the perpendicular to the axis from the point of incidence on any surface, positive when the point of incidence is above the axis.

F the focal length or distance from the 2nd principal point to the intersection of the ray with the axis, positive in the direction of the incident light.

D the thickness or separation between two consecutive surfaces, always positive.

ϕ the angle of incidence.

ψ the angle of refraction.

$\delta = \phi - \psi$ the angle of deviation.

λ the angle at which the refracted ray meets the axis.

The signs of the angles are such that if A and H are positive, then λ is positive; also ϕ is positive for a ray parallel to the axis incident on a surface of which R is positive, if H is positive.

Each surface, or medium, has a suffix which is to be attached to all quantities relating to the surface, and these suffixes increase numerically with the direction of the incident light, commencing with -1 for the medium on the object side of the first surface, which itself has a suffix 0; thus all quantities referring to media such as thicknesses and refractive indices have odd suffixes, while all radii, lengths and quantities referring to surfaces have even suffixes.

The actual formulæ for an object on the axis at an infinite distance are :—

AXIAL RAYS.	EDGE RAYS.	
$\phi_0 = \dfrac{H_0}{R_0}$	$\sin\phi_0 = \dfrac{H_0}{R_0}$	1st surface.
$\psi_0 = \dfrac{\mu-1}{\mu+1}\phi_0$	$\sin\psi_0 = \dfrac{\mu-1}{\mu}\sin\phi_0$	
$\lambda_0 = \delta_0 = \phi_0 - \psi_0$	$\lambda_0 = \delta_0 = \phi_0 - \psi_0$	
$A_0 - R_0 = R_0\dfrac{\psi_0}{\lambda_0}$	$A_0 - R_0 = R_0\dfrac{\sin\psi_0}{\sin\lambda_0}$	
$\phi_2 = \dfrac{(A_0 - R_2 - D_1)\lambda_0}{R_2}$	$\sin\phi_2 = \dfrac{(A_0 - R_2 - D_1)\sin\lambda_0}{R_2}$	2nd and subsequent surfaces.
$\psi_2 = \dfrac{\mu+1}{\mu+3}\phi_2$	$\sin\psi_2 = \dfrac{\mu+1}{\mu+3}\sin\phi_2$	
$\delta_2 = \phi_2 - \psi_2$	$\delta_2 = \phi_2 - \psi_2$	
$\lambda_2 = \delta_2 + \lambda_0$	$\lambda_2 = \delta_2 + \lambda_0$	
$A_2 - R_2 = R_2\dfrac{\psi_2}{\lambda_2}$	$A_2 - R_2 = R_2\dfrac{\sin\psi_2}{\sin\lambda_2}$	

for the last or 2*n*th surface,

$F_{2n} = \dfrac{H_0}{\lambda_{2n}}$	$F_{2n} = \dfrac{H_0}{\sin\lambda_{2n}}$

The practical computer will at once notice that A is determined from A – R, and will not therefore be found very accurately if R is very large; in such a case it is preferable to proceed as below.

If R_{2r} is large, we have—

AXIAL RAYS.	EDGE RAYS.
$H_{2r-2} = R_{2r-2}(\psi_{2r-2} + \lambda_{2r-2})$	$H_{2r-2} = R_{2r-2}\sin(\psi_{2r-2} + \lambda_{2r-2})$
$\phi_{2r} = \dfrac{(A_{2r-2} - R_{2r} - D_{2r-1})\lambda_{2r-2}}{R_{2r}}$	$\sin\phi_{2r} = \dfrac{(A_{2r-2} - R_{2r} - D_{2r-1})\sin\lambda_{2r-2}}{R_{2r}}$
$\psi_{2r} = \dfrac{\mu_{2r-1}}{\mu_{2r+1}}\phi_{2r}$	$\psi_{2r} = \dfrac{\mu_{2r-1}}{\mu_{2r+1}}\sin\phi_{2r}$
$\delta_{2r} = \phi_{2r} - \psi_{2r}$	$\delta_{2r} = \phi_{2r} - \psi_{2r}$
$\lambda_{2r} = \delta_{2r} + \lambda_{2r-2}$	$\lambda_{2r} = \delta_{2r} + \lambda_{2r-2}$
$H_{2r} = H_{2r-2} - D_{2r-1} \times \lambda_{2r-2}$	$H_{2r} = H_{2r-2} - (D_{2r-1} - \Delta D_{2r-2} + \Delta D_{2r})\tan\lambda_{2r-2}$
	where $\Delta D_{2r-2} = 2R_{2r-2}\sin^2\dfrac{\psi_{2r-2} + \lambda_{2r-2}}{2}$
	and $\Delta D_{2r} = 2R_{2r}\sin^2\dfrac{\psi_{2r} + \lambda_{2r}}{2}$
$A_{2r} = \dfrac{H_{2r}}{\lambda_{2r}}$	$A_{2r} = \dfrac{H_{2r}}{\tan\lambda_{2r}} + 2R_{2r}\sin^2\dfrac{\psi_{2r} + \lambda_{2r}}{2}$

The above formulæ will serve for $R_{2r} = \infty$ by using the artifice of substituting $R_{2r} = 100000$ for $R_{2r} = \infty$, which optically would make no appreciable difference to the system, but the following direct method can also be employed :—

$$R_{2r} = \infty$$

AXIAL RAYS.	EDGE RAYS.
$\phi_{2r} = \lambda_{2r-2}$	$\phi_{2r} = \lambda_{2r-2}$
$\lambda_{2r} = \psi_{2r} = \frac{\mu_{2r-1}}{\mu_{2r+1}} \phi_{2r}$	$\sin \lambda_{2r} = \sin \psi_{2r} = \frac{\mu_{2r-1}}{\mu_{2r+1}} \sin \phi_{2r}$
$A_{2r} = \frac{(A_{2r-2} - D_{2r-1})\, \lambda_{2r-2}}{\lambda_{2r}}$	$A_{2r} = \frac{(A_{2r-2} - D_{2r-1}) \tan \lambda_{2r-2}}{\tan \lambda_{2r}}$

As an example of determining the actual aberrations with a view to their subsequent correction, I propose to take the case of a cemented doublet. This cemented doublet was made from Chance Med. Bar. Cr. 1493 and Chance Heavy Flt. 1797, the original data being as follows :—

$$H_o = +\ 15 \qquad \mu^{c}_{-1} = 1{\cdot}00000 \qquad \mu^{F}_{-1} = 1{\cdot}00000$$
$$R_o = +\ 192{\cdot}64$$
$$D_1 = 8{\cdot}75 \qquad \mu^{c}_{+1} = 1{\cdot}57425 \qquad \mu^{F}_{+1} = 1{\cdot}58439$$
$$R_2 = -\ 124{\cdot}46$$
$$D^3 = 4{\cdot}40 \qquad \mu^{c}_{+3} = 1{\cdot}61105 \qquad \mu^{F}_{+3} = 1{\cdot}62779$$
$$R^4 = -\ 2379{\cdot}3$$

All lengths measured in millimetres.

The aperture was thus 30 mm. and the focal length desired was 350 mm. Carried out according to the above data, the focal length was too short and the definition very poor.

It was decided to reduce the thicknesses, which were unnecessarily large, to 4 mm. and 2 mm. respectively and to satisfy conditions (a), (b), and (c).

Altering the thicknesses, the axis ray for C was computed stopping short at the second surface ; to make the focal length 350 mm. it is clear that the last radius must be altered, the new R_4 being found by means of the following :—

$$H_o = 15 \qquad F_4 = 350 \qquad \therefore\ \lambda_4 = \frac{15}{350} \qquad \delta_4 = \lambda_4 - \lambda_2$$

$$\phi_4 = \frac{\delta_4}{1 - \frac{\mu_3}{\mu_5}} \qquad R_4 = \frac{(A_2 - D_3)\, \lambda_2}{\lambda_2 + \phi_4}$$

Putting in the new value for R_4 the calculation was completed, and the focal length found to be 349.992 mm. As five-figure logarithms were used, this result was within the accuracy obtainable by the means employed.

The axis ray for F and the edge ray for C were next computed, the results found being

Axial Rays.		Edge Ray.	
A^C	346.169		
A^F	345.542	A^F	345.500

(N.B.—The formulæ for long radii were used, but the correction terms omitted as being too small to take into account at this stage of the work.)

It will be seen that the chromatic aberration of the axis rays amounts to .63 mm. and is uncompensated, while the spherical aberration is small.

At this stage it is necessary to decide which elements are available to produce alterations in the aberrations and how to allot them to the best advantage.

In this case the only elements available are the two radii R_0 and R_2, as R_4 has already been disposed of in making the focal length 350 mm. It will be best to use R_2 to alter the chromatic aberration, reserving R_0 to deal with the spherical aberration if necessary.

As the chromatic aberration is under corrected, R_2 must be shortened, and it is necessary to determine by what amount.

It can easily be shown that to alter the chromatic aberration by .60 mm., the alteration to be made to λ_2 when computed into air (not into the flint component) is $.60\,\frac{H}{F^2}\,\frac{\nu\nu^1}{\nu-\nu^1}$*where ν, ν^1 are the conventional dispersive constants† of the crown and flint glasses. Applying the formulæ on page 152 we are in a position to find the new $R_2 = -113.467$.

Substituting the new value of R_2, revising the old calculation, and adding the edge ray for C we now find the intersectional distances for the objective having

$R_0 = +\ 192.64$
$R_2 = -\ 113.467$
$R_4 = -3047$

to be

Axial Rays.		Edge Rays.	
A^C	346.200	A^C	346.169
A^F	346.042	A^F	346.177

and this time the correction terms have been included.

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* This is an approximate formula and disregards the thickness of the lenses ; for derivation see Appendix, p. 158.

† $\nu = \frac{\mu_D - 1}{\mu_F - \mu_C}$

The aberrations are :—

CHROMATIC ABERRATION.		SPHERICAL ABERRATION.	
Axis -	.16 mm. uncompensated.	C. -	.02 mm. uncompensated.
Edge -	.01 mm. uncompensated.	F. -	.13 mm. overcompensated.

The values of the focal lengths are :—

AXIAL RAYS.		EDGE RAYS.	
F^C - - -	350.000	F^C - - -	349.861
F^F - - -	349.875	F^F - - -	349.861

their aberrations being :—

CHROMATIC ABERRATION OF THE FOCAL LENGTH.	
Axis - - - -	.12 mm. uncompensated.
Edge - - - -	.00 mm.

ERROR AGAINST THE SINE CONDITION.	
C - - - - -	.14 mm. uncompensated.
F - - - - -	.01 mm. uncompensated.

From the above it is seen that there is still a little uncorrected chromatic aberration, while the spherical aberration is on the whole slightly over corrected. The sine condition is not quite satisfied, but the objective would be good enough for the purpose for which it was intended.

If it were desired to alter the spherical aberration, R_0 would be altered by a small amount, R_2 being also altered to keep the value of λ_2, computed into air, constant, while R_4 would be redetermined to keep the focal length of the whole system 350 mm.

In the case of a triplet system the surfaces would be apportioned according to what aberrations were to be removed, the last radius always being utilised to keep the focal length to its correct value; in the case of a triplet with an air space the surface immediately following the air space would be allotted to the spherical aberration, and the first surface to the fulfilment of the sine condition.

The Testing of Objectives.

Focal Length.

In testing an objective the focal length is first determined. This may be done by one or other of the methods of focal length measurement in general use. It is usually more convenient to measure the intersectional distance, which can be very easily done, and to compare the observed with the computed value already found; if these agree, the focal length will also agree with its computed value.

Spherical Aberration, Centering, Figure.

The objective should be mounted as a telescope with an astronomical eyepiece giving a considerably higher power than that with which it is to be used regularly, and an artificial star at a distance examined, pushing the eyepiece inside and outside the focus.

In both cases the patch of light should be circular (or exactly the same shape as the aperture of the objective) and of uniform brightness. Any want of symmetry in the brightness of the patch of light, either inside or outside the focus, reveals an error in the centering of the objective, while any symmetrical lack of uniformity in the brightness of the patches is caused by spherical aberration.

The appearances caused by undercorrected and overcorrected spherical aberration differ, the former appearing as a hard bright edge to the patch of light inside the focus and a bright centre to the corresponding patch outside the focus, while with the latter the positions are reversed.

Defective figuring of any surface will cause the shape of the patches of light just inside and outside focus to differ from the shape of the aperture.

Coma.

Any really serious departure from the sine condition in an objective of relatively large aperture shows itself as coma, provided the spherical aberration is properly corrected. In looking for coma it is necessary to make sure that the eyepiece is not producing the effect and that the observed coma is really caused by the objective.

Chromatic Aberration.

In testing for chromatic aberration the white artificial star should be replaced by a coloured one, in which the colour is pure and also of known wave-length. Such a coloured artificial star is easily constructed by connecting two cadmium electrodes to the coatings of a small Leyden jar fed from the secondary of an induction coil, and also placing a pinhole just in front of the spark gap. The principal lines of the spark spectrum of cadmium are very bright and well distributed over the whole spectrum; their wave-lengths are well known, and the spacing is such that any line may be recognised rapidly and with certainty.

The coloured star is now viewed through a telescope consisting of the objective to be tested and a high-power eyepiece, the component lenses of which are themselves achromatic.

When a direct vision prism of high dispersion is placed immediately behind the eyepiece and the two together moved inside and outside the focus, the observer can at once see which bands of colour are in focus simultaneously, and can roughly plot the colour curve of the objective, especially if the objective is of considerable focal length, and the focussing tube is provided with a vernier.

Chromatic Aberration of the Focal Lengths.

This aberration very rarely occurs in such magnitude as to be of any importance. Its effect when present to any serious extent in an objective that is properly corrected for chromatic aberration of the intersectional distances is to cause objects of finite area to have images of different sizes, thus producing colour fringes towards the edge of the field of view. It is, of course, essential that the eyepiece used should be thoroughly achromatised as already mentioned in the case of chromatic aberration.

The Hartmann System of Testing.

When extreme accuracy in testing is desired, the Hartmann method of extra-focal measurements should be adopted. This method of testing is so sensitive that there is no telescope objective in existence that will not show aberration when subjected to the following tests.

Spherical Aberration.

A special diaphragm is first prepared, consisting of a card or metal disc arranged to cover up the whole of the aperture and which can be accurately placed over the objective. In this diaphragm a number of small circular holes are made in a series of concentric circles, so that the light passing is restricted to a number of small pencils evenly spaced round a number of zones concentric with the axis of the objective. These pencils should intersect in one point, the focus, if the system is entirely free from spherical aberration over the whole aperture, but in practice they will not do so owing to the residual aberration always present.

A pair of photographic plates are now exposed, one on each side of the focus, and at some distance from it, the resulting negatives after development giving the positions in which the separate pencils of light intersected the two planes in which the exposures were made. If these negatives are measured by means of a travelling microscope, knowing the separation between the two planes, it is easy to calculate the exact points of intersection of the rays of each zone, and so to chart the spherical aberration of the objective over the whole aperture.

Chromatic Aberration.

When the colour curve of an objective is required a modification of the above method can be employed. A diaphragm is again fitted to the objective but with only two apertures diametrically opposite and each distant from the axis by two-thirds of the semi-aperture. A prism is now placed in front of the objective with its refracting edge parallel to the line joining the centres of the two apertures.

Two negatives are taken as before, using the cadmium spark as a light source, and the negatives measured. The numerical reduction proceeds on similar lines, and gives the differences of the intersectional distances for the wave-lengths of the spectral lines utilised, or, in other words, the colour curve of the objective.

Conclusion.

Owing to the necessity of restricting the length of this paper, it has only been possible to give a brief outline of this interesting branch of optics. Its purpose is not to be a complete exposition of the subject, but to serve as a starting point from which those who would like to follow up the mathematical side of the problem, but are as yet unacquainted with it, may be led to a larger interest in the subject. The two most valuable books dealing with the telescope objective are Steinheil and Voit's "Handbuch der Angewandten Optik" and Gleichen's "Lehrbuch der Geometrischen Optik." Unfortunately these two books are in the German language, but I believe that they are in course of being translated into English. With these two books, the tables published by Messrs. Smith & Cheshire, a glass catalogue, and a table of logarithms, the designer has all the material required to work out the final curves, substances and residual aberrations of any telescope objective he may desire.

APPENDIX.

A method of finding the alteration required to vary the chromatic aberration by any desired small amount.

Let F_C, F_D, F_F be the focal lengths of the combination for the C, D and F lines of the spectrum; also let ϕ be the power of the crown component, K and K′ the total curvatures of the crown and flint components, n and n' with suitable suffixes their refractive indices.

Consider now an objective suffering from positive chromatic aberration (longitudinal) A.

We have
$$\frac{1}{F_F} = K\,(n_F - 1) + K'\,(n'_F - 1)$$
$$\frac{1}{F_C} = K\,(n_C - 1) + K'\,(n'_C - 1).$$

By subtraction $\frac{1}{F_F} - \frac{1}{F_C} = K\,\Delta + K'\Delta'$, where $\Delta = n_F - n_C$ and $\Delta' = n'_F - n'_C$

also $\frac{1}{F_F} - \frac{1}{F_C} = \frac{F_C - F_F}{F_F\,F_C} = \frac{A}{F_F\,F_C} = \frac{A}{F^2}$ very nearly.

Now add increases of curvature δK and $\delta K'$ such that

$$0 = \delta K\,(n_D - 1) + \delta K'\,(n'_D - 1) \text{ and } -\frac{A}{F^2} = \Delta\,\delta K + \Delta\,\delta K'$$

i.e., we have increased the powers of the lenses without altering the focal length of the combination and have at the same time introduced a chromatic aberration equal to A but of opposite sign.

We have immediately

$$\delta K' = -\frac{\delta K\,(n'_D - 1)}{n'_D - 1}$$

and consequently

$$-\frac{A}{F^2} = \Delta\,\delta K - \Delta'\delta K\,\frac{n \quad - 1}{n'_D - 1}$$

But

$$\delta K = \frac{\delta\phi}{n_D - 1}$$

$$\therefore\ -\frac{A}{F^2} = \Delta\,\frac{\delta\phi}{n_D - 1} - \Delta'\,\frac{\delta\phi}{n'_D - 1}$$

Now

$$\Delta = \frac{n_D - 1}{\nu} \text{ and } \Delta' = \frac{n'_D - 1}{\nu'}$$

$$\therefore\ -\frac{A}{F^2} = \frac{\delta\phi}{\nu} - \frac{\delta\phi}{\nu'} = \delta\phi\left(\frac{1}{\nu} - \frac{1}{\nu'}\right) = -\,\delta\phi\,\frac{\nu - \nu'}{\nu\,\nu'}$$

$$\therefore\ \delta\phi = \frac{A}{F^2}\,\frac{\nu\,\nu'}{\nu - \nu'}$$

By the axial trigonometrical formulæ $\delta\beta = H\,\delta\phi$.

∴ the alteration to the β of the crown lens to correct a chromatic aberration of the whole system of the amount A is

$$A \cdot \frac{H}{F^2} \cdot \frac{\nu\,\nu'}{\nu - \nu'}.$$

P. F. E.

Principles and Methods of Calculating Telescope Objectives.

By T. SMITH, B.A.

THE President has asked me to give an account of the method of calculation employed at the National Physical Laboratory in deriving the curvatures and radii contained in the booklet "Constructional Data of Small Telescope Objectives." The essential formulæ have been published in a series of papers read before the Physical Society, and reprinted by Harrison & Sons as an extract from the thirteenth volume of the "National Physical Laboratory Collected Researches" entitled "Papers on Optical Calculations and Measurements relating to Lenses and Lens Systems"; it may be obtained from Harrison's for three shillings. Anyone desiring to obtain fuller particulars than are given here should consult this booklet. My intention on the present occasion is to give a short account of the method in question which will enable anyone to solve problems in the design of a simple cemented doublet lens while making no greater demands on his mathematical equipment than ability to solve a quadratic equation.

We are all familiar with the properties of lenses as they are described in books on Geometrical Optics, and realise how greatly our conception of these properties has been simplified by their general reference by Gauss to four fundamental points on the lens axis, quite irrespectively of the number and position of the surfaces and other characteristics of the concrete system of lenses itself. In the systems with which we ordinarily have to deal these properties are fully determined when the positions of the two principal foci and the magnitude of the focal length are known. The position occupied by the lens so far as this theory goes is quite accidental and without effect on the way in which paraxial pencils of rays are refracted. Thus, whatever the position of the object may be relative to the lens, only three quantities are required to find the magnitude and position of its image; one of these quantities is the focal length of the system or its reciprocal the dioptric power, and the other two are the distances of the principal* points or of the principal foci from the external surfaces of the lens.

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* The term principal point is frequently used for unit point. The use of two technical terms with no difference of meaning is to be deprecated, and the more descriptive term unit point is to be preferred when this is its one general property. It would be a great convenience to use unit point when the external rerfactive indices differ, and reserve principal point for the combined unit-nodal point where the external media are the same.

In the very important class of lenses we call "thin," where the thicknesses of all the components are so small that we can regard all the refracting surfaces as intersecting the axis at the same point, the two unit points coincide with one another and with this point in which the axis meets the lenses. In this case, therefore, so long as the external media are the same only one quantity, the focal length, or alternatively the power, is needed to specify completely the manner in which paraxial rays will be refracted. It will presently be seen that there are corresponding simplifications in the aberrations of such lenses.

It has so far been assumed that light of one colour only is considered. For every different wave length there will be a different focal length and definite positions for the principal points. From the statements already made it is clear that in the general case with identical external media three conditions have to be satisfied to ensure paraxial rays behaving alike for light of two different colours. With thin lenses on the other hand the positions of both principal points, no matter to what colour they relate, are determined by the intersection of the lens surfaces with the axis; the conditions that the principal points should occupy the same positions for different colours are therefore automatically fulfilled, and complete colour correction for paraxial rays is secured by satisfying the one condition of equality for the focal lengths corresponding to these colours. Though no actual lenses are "thin" in the mathematical sense, many lenses approximate sufficiently to it to be perfectly corrected for colour for all practical purposes when only one condition is satisfied.

The Gauss theory is only strictly true of rays which are always close to the axis, so that both the aperture of the lens and also all objects and images must be regarded as extending only to a short distance from the lens axis. As a result of this, though we are justified in saying that the image of a small plane object placed normally to the axis is itself normal to the axis, we cannot say that this image is plane. We simply know that if not plane it has finite curvature, but what that curvature is the Gauss theory is not able to state. In particular we are not able to say that the "principal planes" are plane; to be exact, we should speak of principal surfaces which may for paraxial rays be replaced by their tangent planes at the points where they meet the lens axis. The curvature of the principal surfaces has, in fact, a very important bearing on the correction of lens aberrations.

To secure a reasonable amount of light and a suitable field of view, the region for which the Gauss theory holds must always be exceeded. As these limits are exceeded divergencies from the Gauss laws will begin to appear; the further the departure from the limits the more obvious will the differences become, and the general tendency will be for the rate at which the departures grow to be rapidly accelerated as the apertures and field of view are more and more increased. One method of regarding such phenomena is to suppose that the way in which a ray is refracted may be determined by a number of different characteristics of the particular lens system, of which only one, the one resulting in the Gauss laws, exercises perceptible influence on paraxial rays, a second starting to exercise a moderating influence as soon as the region for which the Gauss laws are strictly true is exceeded, a third term of the series becoming appreciable at larger apertures and fields, a fourth at still greater values, and so on. The method to be described is of this character. It will not be difficult to realise that the system must be applied to a considerable number of lenses before we can say how many of the successive terms must be taken to give a result of definite accuracy with a given aperture and field of view. The utility of the method depends entirely upon its capacity to give results within the required degree of accuracy with less trouble than is involved in tracing rays trigonometrically through the lens system.

To obtain a satisfactory image of any object the rays starting from any one point of the object should be so refracted that they will pass through the same image point, no matter at what aperture they meet the lens. The image point for paraxial rays passing through a thin lens is determined when the focal length is given, and thus to secure good correction rays through other zones must be made to give an image of the same size and in the same position as that formed by paraxial rays. It is the necessity for thus causing all zones of a well corrected system to conform to the paraxial zone which gives to the Gauss laws their importance. When the outer zones conform with the paraxial zone the terms in the series before mentioned which constitute departures of various kinds from the paraxial laws must each be equal to zero. If it happens that they cannot all be made equal to zero, it is necessary to consider what compromise is the most suitable to adopt. If the field of view is only required to be of about the size to which paraxial laws apply, but the aperture of the lens is to be increased to an extent within which the modifications represented by the second term of the series are important, though the alterations represented by the third term are still imperceptible, evidently the proper course to adopt is to make the second term vanish. If on the other hand the aperture is sufficiently large for the influence of the third term to be appreciable, and it is impossible to make the second and third terms vanish simultaneously with the type of lens proposed, a balance has to be struck between the two terms so as to get the least possible departure from the paraxial laws when the whole of the zones are taken into account. If the best compromise that can be made is not sufficiently good it becomes necessary to consider a lens system of a different kind where there is more freedom, such as may be obtained by using two separated lenses instead of one single lens, or employing a triple lens instead of a double one, or departing from spherical surfaces. In the case of telescope objectives, which are very nearly thin lenses so far as their design is concerned, it is found that the second and third modifying terms have to be taken into account, while the rest may be neglected in nearly all cases ; that the second and third will not vanish simultaneously for spherical surfaces, but that a good compromise is obtained when the second very nearly vanishes. The modifications in the refraction of rays due to the presence of the second term are called first order aberrations ; those due to the presence of the third term second order aberrations ; and so on. Experience then shows that for a telescope objective the best form is very close to that obtained by eliminating the first order aberrations, and the calculation of thin objectives free from first order aberrations is thus the basis of the design of a telescope objective.

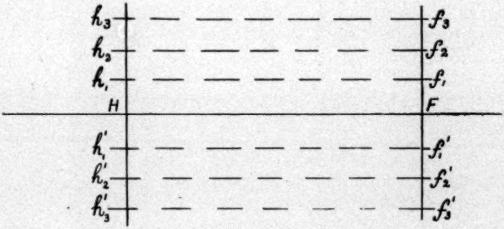

Fig. *a*.

In Fig. *a*, if HF is the axis, H the principal point, and F the principal focus, we may suppose that h_1 h'_1 represents the aperture and f_1 f'_1 the field for which the Gauss laws are practically exactly true ; h_2 h'_2 the aperture and f_2 f_2 the field for which the Gauss laws together with the first order aberrations may be regarded as quite correct ; h_3 h'_3 the aperture, and f_3 f'_3 the field within which the Gauss laws together with both the first and second order aberrations account adequately for the refraction of rays. According to the foregoing explanation a telescope objective which will usually have a full aperture about equal to h_3 h'_3 and a field represented by f_2 f'_2 should have a form almost exactly agreeing with that necessary to give the best results with the aperture h_2 h'_2 on the field f_1 f'_1 and with the aperture h_1 h'_1 on the field f_2 f'_2. The Gauss laws tell us that the definition over the field f_1 f'_1 from the aperture h_1 h'_1 is practically perfect ; when the field is extended to f_2 f'_2 with the same aperture it is found that there is no distortion present in the image, but that both curvature and astigmatism are present in definite amounts which we are unable to control. If HF be taken as the unit of length instead of obtaining a point to point correspondence in the image of a point there are two focal lines, one pointing towards the axis and the other in a direction perpendicular to this, and the centres of these lines instead of lying in the plane through F perpendicular to the axis lie on spheres of curvatures $1 + \varpi$ and $3 + \varpi$ respectively where ϖ is the Petzval sum. In telescope objectives ϖ necessarily lies between about .68 and .75, so that it is impossible to remove the very appreciable curvature in the neighbourhood of the axis of the surfaces on which the centres of the focal lines lie. The only way, then, in which one objective will differ from another when light of one colour alone is considered will depend upon the degree to which light from the aperture $h_2\,h'_2$ is made to give an image identical in position and size with that from the aperture $h_1\,h'_1$. The two aberrations which lead to departures from the Gauss images with this increased aperture are first order spherical aberration and first order coma. The coma of a thin lens system is intimately connected with the curvature of the principal surfaces, and what is in fact done to eliminate coma is to give the proper curvature to these surfaces. It may, in the first place, be noted that there is justification for speaking of a surface through a principal point having a surface as its image. For owing to the lens surfaces being coincident the principal points are free from spherical aberration. The sine condition also is satisfied, and there is, therefore, no coma. There is obviously in the image of a point of the surface off the axis no difference between the refraction of rays not in a principal plane and those lying in such a plane; in other words, there is no astigmatism: there may thus only be a difference in the curvature of the object surface and its image, and distortion. Since the magnification in the neighbourhood of the principal point is unity in all directions, the difference in curvature of the object and image surfaces will be independent of the curvature of the object surface. It is easily seen that the curvature—taken positive when the surface is convex to the incident light considered—of a surface through the first principal point exceeds that through the second principal point by ϖ. The two surfaces through the principal points which have this difference in curvature and are free from distortion, so that an incident ray meeting one surface after refraction intersects the other at the same distance from the axis and in the same principal plane, are the principal surfaces. These results are readily seen to hold for a single thin lens. Fig. *b* shows a plano-convex lens. The curved refracting surface is obviously one of the principal surfaces. Let PN, a line parallel to the lens axis, meet this surface in P and the plane surface in N. Any ray through P will, after refraction, diverge from P′ where P′ is the point on PN such that $PN = \mu P'N$. In other words, the curvature of the second principal surface is $\frac{1}{\mu}$ times that of the first. Since the focal length is unity, the curvature of the first surface is $\frac{1}{\mu - 1}$, and that of the second is therefore $\frac{1}{\mu(\mu - 1)}$, and the curvature of the first minus that of the second is

$$\frac{1}{\mu - 1} - \frac{1}{\mu(\mu - 1)} = \frac{1}{\mu} = \varpi$$

Evidently in the case of any thin single lens the principal surfaces may be found by drawing the spheres which pass through H, H′ the principal points, and I, I′ the circle in which the two surfaces of the lens intersect (Fig. *c*).

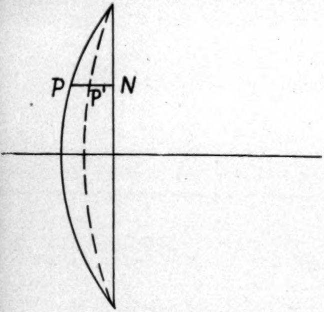

Fig. *b*.

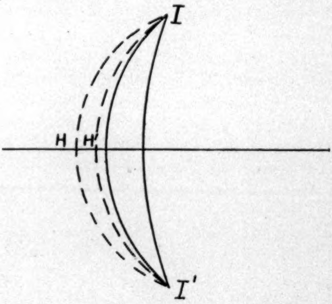

Fig. *c*.

A single thin lens of unit power may be made with surfaces of varying power: for instance, each may be of power $+\frac{1}{2}$. In this case symmetry suffices to show that the curvatures of the principal surfaces will be equal and opposite, and therefore equal respectively to $+\frac{1}{2}\varpi$ and $-\frac{1}{2}\varpi$. In the convexo-plane lens the powers are 1 and 0, and the curvatures of the principal surfaces are $\frac{1}{\mu - 1}$ and $\frac{1}{\mu - 1} - \varpi$. When the lens becomes meniscus as in Fig. *c*, the system tends to throw the principal points on the convex side of the glass surfaces, an effect of course corresponding with the advanced positions of the principal points in telephoto systems where a positive combination is followed at some distance by a negative one. It is evident that the curvature of the principal surfaces is altering more rapidly than the general shape of the lens. Instead of considering the powers of the lens surfaces consider their actual curvatures. The power of the lens remains constant if the difference between the curvatures of the surfaces is kept the same; that is to say the same addition to the curvature should be made for each. If the curvatures of the lens surfaces are R_1 and R_2, it is not difficult to see that those of the principal surfaces are $R_1 + \frac{R_2}{\mu}$ and $\frac{R_1}{\mu} + R_2$ respectively. If R_1 and R_2 are each increased by the amount r, the increase in the curvature of each principal surface is therefore $r\left(1 + \frac{1}{\mu}\right)$. A simple extension of this result is that in any thin lens, no matter of how many components it may consist, an increase in the curvature of every refracting surface by r will increase the curvature of the principal surfaces by $r(1 + \varpi)$, but will make no difference in the power of any component lens or in the power of the complete system. Thus by altering the curvature of every surface by the same amount the focal length of the system and the chromatic variation of the focal length will be unaltered, while any required addition can be made to the curvature of the principal surfaces.

The next result is one that must be taken for granted. It is a particular case of a general theorem which gives the exact amount of coma for any zone of any lens when a ray has been traced through this zone of the system from the axial point of the object.* The sine condition implies that the coma can be found from such a ray, but I think it has not so far been recognised that the actual amount of coma, eliminated from spherical aberration and any departure from the sine condition, is given by such a ray. In the present case the result referred to may be stated in the form that the amount of coma (first order) is determined, whether spherical aberration be present or not, by the curvature of the principal surfaces. Therefore, to determine the correct curvature of these surfaces we may consider the case when spherical aberration and coma are both absent and the sine condition is therefore satisfied.

When the object is at infinity the second principal surface must obviously be the sphere through the second principal point which has its centre at the principal focus, as shown in Fig. *d*. The dotted curve gives the first principal surface: the curvatures of the two surfaces in this case will evidently be $1 + \varpi$ and 1. When the object is not at infinity the case is not quite so simple. In Fig. *e* let O be the point in which the plane of the object meets the axis, and let I be the image of O formed by paraxial rays. Let a ray from O meet the sphere centre O which passes through H in p, and the first principal surface in h. Let the corresponding refracted ray meet the second principal surface in h' and the sphere through H′ centre I in p'. Then, from the definition of principal surfaces

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* For a proof and fuller information see "The Determination of Coma from a Central Ray." Proc. Phys. Soc. Vol. XXIX, p. 293.

$h\,h'$ is parallel to the axis, and when there is freedom from spherical aberration and the sine condition is satisfied $p\,p'$ will be parallel to the axis. The curvatures of the principal surfaces which will give freedom from coma are therefore those which will make $p\,p'$ parallel to the axis. The principal surfaces cannot be made to coincide with the spheres p H and p' H′ respectively, since the difference between these curvatures is -1 and the corresponding difference of curvature between the principal surfaces is $+\varpi$. As the rays are not inclined at large angles to the axis $p\,h$ and $h'\,p'$ will only differ slightly from $p\,q$ and $q'\,p'$ where q and q' are the points in which $p\,p'$ meets the principal surfaces. The condition that $p\,p'$ should be parallel to the axis is $p\,q : q'\,p' = \mathrm{OH} : \mathrm{H'I}$. Also $p\,q$ and $q'\,p'$ are proportional to the differences in the curvatures of the sphere and principal surface in the object region on the one hand and between the sphere and principal surface in the image region on the other hand. Calling σ and σ' the curvatures of the spheres and ρ and ρ' the curvatures of the principal surfaces, so that

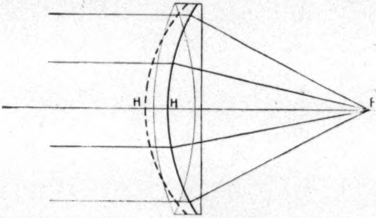

Fig. *d*.

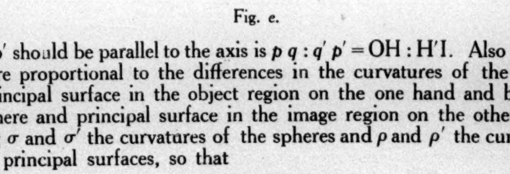

Fig. *e*.

$$\sigma' = \sigma + 1$$

$$\rho = \rho' + \varpi$$

and $$\text{magnification} = \frac{\mathrm{H'I}}{\mathrm{HO}} = \frac{\sigma}{\sigma'},$$

the condition for freedom from coma in the image is

$$\frac{\rho' - \sigma'}{\rho - \sigma} = -\frac{q'p'}{pq} = -\frac{\mathrm{H'I}}{\mathrm{OH}} = \frac{\sigma}{\sigma'}$$

$$= \text{magnification.}$$

If m be written for the magnification, since

$$\sigma = \frac{m}{1 - m}$$

$$\sigma' = \frac{1}{1 - m}$$

the curvatures of the principal surfaces must be

$$\rho = \sigma + \sigma' + \sigma' \varpi = \frac{1 + m + \varpi}{1 - m}$$

$$\rho' = \sigma + \sigma' + \sigma \varpi = \frac{1 + m + m\varpi}{1 - m}.$$

An important result which follows from the preceding theory is that if the curvatures which secure freedom from coma for two magnifications with given glasses are known, those which will give freedom from coma for some other magnification are found by simple proportion. For example, Groups D and G of the "Constructional Data of Small Telescope Objectives," give the curvatures for freedom from coma with an object at infinity with crown leading and with flint leading respectively. When the glasses are Crown $\mu_{\mathrm{D}} = 1{\cdot}5175$ and Flint $\mu_{\mathrm{D}} = 1{\cdot}6225$ the curvatures are as follows :—

	Outer Crown Surface.	Cemented Surface.	Outer Flint Surface.
Group D ...	1·558	3·214	0·853
Group G ...	– 0·023	4·795	2·434

The curvatures for an objective of these glasses corrected for coma for unit magnification ($m = -1$) will be the means of the above, *i.e.*,

$$\frac{1 \times 1{\cdot}558 + 1 \times (-0{\cdot}023)}{1 + 1} = 0{\cdot}768$$

$$\frac{1 \times 3{\cdot}214 + 1 \times 4{\cdot}795}{1 + 1} = 4{\cdot}005$$

$$\frac{1 \times 0{\cdot}853 + 1 \times 2{\cdot}434}{1 + 1} = 1{\cdot}644$$

If as another example the image is to be $\frac{3}{7}$ the size of the object ($m = -\frac{3}{7}$) with the crown lens towards the object, the curvatures will be

$$\frac{7 \times 1{\cdot}558 + 3 \times (-0{\cdot}023)}{7 + 3} = 1{\cdot}084$$

$$\frac{7 \times 3{\cdot}214 + 3 \times 4{\cdot}795}{7 + 3} = 3{\cdot}688$$

$$\frac{7 \times 0{\cdot}853 + 3 \times 2{\cdot}434}{7 + 3} = 1{\cdot}327$$

Another way in which these results could be reached is to note that Groups D and G differ in the curvatures of corresponding surfaces with these glasses by 1·581. The change from the crown position in the first illustration is half this, and in the second illustration $\frac{3}{10}$ of this: *i.e.*, the differences from Group D are 0·791 and 0·474, the direction of the change in each surface being the same as in going from group D to group G.

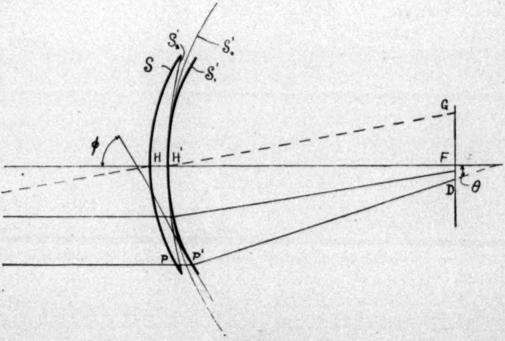

Fig. *f*.

It might be expected from the foregoing explanation of the means of correcting coma in thin lenses that when coma is present the departure of the principal surface from its proper curvature will give the amount of coma present, and such is in fact the case. The result is of particular importance in connection with rays traced through a system from the centre of the object. Such rays are primarily traced through a system to determine the amount of spherical aberration present. Consider the case when the object is at infinity. Let a ray PP′ (Fig. *f*) parallel to the axis meet the refracted ray P′D in P′. If S_0' is the position for the second principal surface corresponding to no coma, and S_1' is the actual principal surface, the amount of coma (first order) will be proportional to the difference in the curvatures of the surfaces S_0' and S_1', the coma being inward when the curvature of S_1' exceeds that of S_0', *i.e.*, when P′ is nearer to the focal plane than it should be. Whatever the order of the coma may be, it can be measured when S_1' has been found. For let the refracted ray meet the focal plane in D. Then rays in a secondary plane meeting the principal surfaces in the zone containing P and P′ will have a comatic displacement in the image plane relative to G, the point of intersection with the ray through the principal points, equal to

$$(\mathrm{P'D} - \mathrm{H'F}) \times \frac{\mathrm{FG}}{\mathrm{H'F}}.$$

If the angle made by P′D with the axis is θ, the corresponding displacement for rays in a primary plane is

$$\left\{\left(1 + \tan\theta \frac{d}{d\theta}\right)\mathrm{P'D} - \mathrm{H'F}\right\} \times \frac{\mathrm{FG}}{\mathrm{H'F}}.$$

The rays from the complete zone containing P and P′ meet the image plane in a locus which is fully determined by the spherical aberration and these two displacements. This explanation of coma and of the means of measuring it and of removing it has been given at some length because it is not so easy an aberration to grasp and to see the correct way of removing as most others. It is hoped that a clearer understanding of it may now be assumed.

I pass now to the formulæ used in calculating the objective. The crown and flint glasses will be supposed to have refractive indices for the D line equal to μ and μ' respectively; their ν values will be denoted by ν_C and ν_F.

The condition for freedom from colour is satisfied if the powers of the crown and flint components for unit focal length of the complete lens are given by $K = \frac{\nu_C}{\nu_C - \nu_F}$ and $K' = \frac{-\nu_F}{\nu_C - \nu_F}$. The total curvatures of these components are R and R′ where $R = \frac{K}{\mu - 1}$ and $R' = \frac{K'}{\mu' - 1}$. The Petzval sum ϖ is equal to $\frac{K}{\mu} + \frac{K'}{\mu'}$. It will thus be seen that K, K′, R, R′, ϖ are independent of the general inward or outward bulge of the lens, *i.e.*, of the shape of the principal surfaces. Now just as there are simplifications in the Gauss Theory for a system that is thin, so are there simplifications in the aberration theory. As a rule six quantities, A, B,

C, ϖ, B', A' are necessary to determine fully all first order aberrations. When the lens is thin the relations

$$A - 2B + C = 1,\ B - 2C + B' = \varpi,\ C - 2B' + A' = 1$$

are satisfied and it is therefore only necessary to find two of these quantities in addition to ϖ. These other quantities depend on the shape of the principal surfaces. Let it be supposed that the curvature of the cemented surface is to be r. Then the curvature of the outside surface of the crown lens will be $R + r$ and that of the outer surface of the flint lens will be $r - R'$.

The value of $C - B$ is then

$$K(R + R' - \varpi) - R' + (1 + \varpi)\,r.$$

From what has already been said it is clear that $(1 + \varpi)\,r$ is the increase in the curvature of the principal surfaces compared with the lens having $r = 0$.

The condition for freedom from coma with the crown component leading is $A - B = 0$, or for a thin lens $C - B = 1$, when the object is at infinity; thus there is a linear equation for r. When the flint component leads the corresponding condition is $A' - B' = 0$, reducing to $C - B = -(1 + \varpi)$ for a thin lens. For finding the spherical aberration a formula for C is used. It may be written—

$$\begin{aligned} C = {} & K R^2 + K' R'^2 + 2 K K' (R + R' - \varpi) \\ & + [3\{K(R + R' - \varpi) - R'\} + (1 + K')\,\varpi]\,r + (1 + 2\varpi)\,r^2 \end{aligned}$$

The quantity inside the brackets { } has already been found.

The condition for freedom from spherical aberration with the crown component leading is

$$A = 0 \text{ or } 1 - 2(C - B) + C = 0$$

and that for the flint component first is

$$A' = 0 \text{ or } 1 + 2\varpi + 2(C - B) + C = 0$$

these being quadratic equations. To illustrate the application of these equations an example may be taken: quite imaginary glasses have been chosen which do not involve many figures, so that they are easy to follow. Let $\nu_C = 55$, $\nu_F = 30$. Then for freedom from colour

$$K = \frac{55}{55 - 30} = 2{\cdot}2$$

$$K' = \frac{-30}{55 - 30} = -1{\cdot}2$$

Take also $\mu = 1\cdot5,\ \mu' = 1\cdot8$

Then
$$R = \frac{2\cdot2}{0\cdot5} = 4\cdot4$$
$$R' = \frac{-1\cdot2}{0\cdot8} = -1\cdot5$$
$$\varpi = \frac{2\cdot2}{1\cdot5} - \frac{1\cdot2}{1\cdot8} = 0\cdot8$$

so that
$$R + R' - \varpi = 2\cdot1$$

Referring to the formulæ for C − B
$$\begin{aligned} C - B &= 2\cdot2 \times 2\cdot1 - (-1\cdot5) + (1 + 0\cdot8)\,r \\ &= 6\cdot12 + 1\cdot8\,r \end{aligned}$$

For lenses with No Coma

Crown leading $r = -\frac{6\cdot12 - 1}{1\cdot8} - = -\frac{5\cdot12}{1\cdot8} = -2\cdot84$

(Group D)

Curvatures $\begin{cases} R + r = 1\cdot\dot{5} \\ \quad r = -2\cdot8\dot{4} \\ r - R' = -1\cdot3\dot{4} \end{cases}$

Flint leading $r = -\left(\frac{6\cdot12}{1\cdot8} + 1\right) = -4\cdot4$

(Group G)

Curvatures $\begin{cases} R + r = 0 \\ \quad r = -4\cdot4 \\ r - R' = -2\cdot9 \end{cases}$

Now turn to the formula for C
$$\begin{aligned} K R^2 &= 42\cdot592 \\ K' R'^2 &= -\ 2\cdot7 \\ 2 K K' (R + R' - \varpi) &= -\ 11\cdot088 \\ &\ \overline{\ 28\cdot804\ } \end{aligned}$$

and therefore
$$\begin{aligned} C &= 28\cdot804 + [3 \times 6\cdot12 + (1 - 1\cdot2) \times 0\cdot8]\,r + (1 + 2 \times 0\cdot8)\,r^2 \\ &= 28\cdot804 + 18\cdot2\,r + 2\cdot6\,r^2 \end{aligned}$$

so that
$$\begin{aligned} A &= 1 - 2 \times 6\cdot12 + 28\cdot804 + (-2 \times 1\cdot8 + 18\cdot2)\,r + 2\cdot6\,r^2 \\ &= 17\cdot564 + 14\cdot6\,r + 2\cdot6\,r^2 \end{aligned}$$

and
$$\begin{aligned} A' &= 2\cdot6 + 2 \times 6\cdot12 + 28\cdot804 + (2 \times 1\cdot8 + 18\cdot2)\,r + 2\cdot6\,r^2 \\ &= 43\cdot644 + 21\cdot8\,r + 2\cdot6\,r^2 \end{aligned}$$

So for lenses with No SPHERICAL ABERRATION.

Crown leading $\mathrm{A} = 0$, i.e. $r = \dfrac{-7{\cdot}3 \pm \sqrt{7{\cdot}6236}}{2{\cdot}6}$

or

(Group A) $r = -1{\cdot}7457$
(Group B) $r = -3{\cdot}8696$

Flint leading $\mathrm{A}' = 0$, i.e. $r = \dfrac{-10{\cdot}9 \pm \sqrt{5{\cdot}3356}}{2{\cdot}6}$

or

(Group E) $r = -3{\cdot}3039$
(Group F) $r = -5{\cdot}0807$

Suppose now that instead of making the lens achromatic it had been desired to leave a definite amount of chromatic aberration. The definition of the ν of any glass is

$$\nu = \frac{\mu_D - 1}{\mu_F - \mu_C}$$

where μ_C, μ_D, μ_F are the refractive indices for the C, D and F lines. If then r is the curvature of any surface separating air and this glass the powers of this surface for these three wave lengths will be

$$\begin{aligned} \kappa_C &= r\,(\mu_C - 1) \\ \kappa_D &= r\,(\mu_D - 1) \\ \kappa_F &= r\,(\mu_F - 1) \end{aligned}$$

and, therefore,

$$\begin{aligned} \kappa_F - \kappa_C &= r(\mu^{F} - \mu_C) = \frac{1}{\nu}\,r\,(\mu_D - 1) \\ &= \frac{1}{\nu}\,\kappa_D \end{aligned}$$

For a thin lens powers are simply added, and therefore for the complete system

$$\kappa_F - \kappa_C = \sum \frac{\kappa_D}{\nu}$$

where on the right κ_D is the total power of the lens made of the glass to which ν refers. In the case of two thin components where the power is unity for the D line, the difference in the powers for the C and F lines is, in the previous notation

$$\frac{\mathrm{K}}{\nu_C} + \frac{\mathrm{K}'}{\nu_F}$$

and in accordance with the ν notation for a single glass $\kappa_F - \kappa_C$ for the complete lens may be called $\frac{1}{\nu}\kappa_D = \frac{1}{\nu}$, so that the equations giving the powers of the components for this difference in the powers for the C and F lines are

$$K = \frac{\nu_C}{\nu} \cdot \frac{\nu - \nu_F}{\nu_C - \nu_F}$$

$$K' = \frac{\nu_F}{\nu} \cdot \frac{\nu_C - \nu}{\nu_C - \nu_F}$$

and can thus be written down as before. These new values of K and K′ simply take the place of those used in the previous case; the rest of the calculations follow exactly the same course as before, starting from the new values of K and K′ and the known values of μ and μ'. Suppose, for example, with the previous glasses it were desired to make the power for the C line exceed that for the F line by ·05, the power for the D line being unity. Here $\nu = -20$, and

$$K = \frac{55}{-20} \cdot \frac{-20-30}{55-30} = 5{\cdot}5,\ R = \frac{5{\cdot}5}{0{\cdot}5} = 11,$$

$$K' = \frac{30}{-20} \cdot \frac{55-(-20)}{55-30} = -4{\cdot}5,\ R' = \frac{-4{\cdot}5}{0{\cdot}8} = -5{\cdot}625, \text{ etc.}$$

This, of course, would be a very extreme case of overcorrection. It may be noted that in this notation a positive ν corresponds to undercorrection, and a negative ν to overcorrection for colour.

Owing to the non-proportionality of the dispersions for crown and flint glasses, correction for colour really involves making the power a maximum for some chosen wave length. The differences in power for wave lengths sensibly different from the maximum will be practically proportional to the square of the dispersion differences. Thus in a thin lens when K and K′ are given the values secured by the formulæ illustrated above, the maximum power will relate to a wave-length between D and F because $\mu_D - \mu_C$ is less than $\mu_F - \mu_D$. It might be thought that in consequence the method illustrated would not give satisfactory correction. It happens, however, that the introduction of the standard thicknesses without altering the curvatures gives just about the change in the focal curve needed for good correction, so that generally speaking where considerable exactitude is not required, the desirable type of colour correction is secured by proceeding in the above way. When the introduction of whatever thicknesses are to be used does not give the type of colour correction that is wanted, the required alterations in the curves are obtained at once by finding the value of ν to which the changes correspond and using the generalised equations to determine K and K′.

The introduction of thicknesses has other effects. In the first place the power of the complete lens will differ slightly from unity. This may at once be corrected by altering all the curvatures in the same constant ratio as is necessary to make the power of the complete lens unity, the thicknesses being kept unchanged. If very great exactitude is wanted in the focal length the procedure can be slightly varied. Let the introduction of the thicknesses change the power from 1 to $1 - \epsilon$. Since the thicknesses are small the introduction of thicknesses x times as great will make the power $1 - x\epsilon$. If now the linear dimensions of the whole system are made $1/x$ times their former value, the thicknesses will resume the values required and the power of the system will be increased x times. Its value is required to be equal to unity. The equation for x, the factor by which all the curvatures are to be multiplied, is therefore given by

$$1 = x(1 - x\varepsilon) \text{ or } 1 - x + \varepsilon x^2 = 0$$

The solution expressed as a series is

$$x = 1 + \varepsilon + 2\varepsilon^2 + 5\varepsilon^3 + 14\varepsilon^4 + \ldots$$

In practical cases ϵ is so small that it is hardly ever necessary to take any notice of any term after $2\epsilon^2$.

The introduction of thicknesses also alters the aberration coefficients, and thus disturbs the corrections that have been made. Generally speaking, with a thin objective the alteration in the coma is too small to be of any account. In an objective corrected for spherical aberration, say one for which $A = 0$ when the thicknesses are zero, the introduction of thicknesses makes the objective undercorrected (A positive). This, however, is exactly what is required since there is always present spherical aberration of the second order tending to overcorrect the objective. Whether the amount of undercorrection due to the thicknesses is just about what is required to compensate for the overcorrection of the second order aberrations depends upon what the aperture is to be. Generally speaking, the result is about right up to moderately large relative apertures, while the undercorrection is not enough for very large relative apertures. In these cases the equation to be solved is not $A = 0$ but $A =$ some positive quantity which can be easily determined with any glasses by finding the error involved when the objective has been solved for $A = 0$. A few calculations with actual objectives of various glasses will enable a suitable value of A to be assumed at once.

The formulæ that have now to be given are suitable for the calculation of the aberration coefficients when the thicknesses are taken into account. The notation used is explained in a paper on optical calculations, appearing in Vol. XIII. of the "Transactions of the Optical Society," page 196. Let it be supposed that the number of surfaces in the system is n, numbered $1, 2, \ldots p, \ldots n-1, n$, and that the power of the system is unity. As a preliminary the power will have been calculated from each end of the system. The curvature of the p^{th} surface will be denoted by R_p, so that $\mathrm{R}_p \cdot r_p = 1$. Put

$$k_p = \frac{\partial\,\kappa_{1,p}}{\partial\,\kappa_p}\left(\frac{\kappa_{1,p}}{\mu_p{}^2} - \frac{\kappa_{1,p-1}}{\mu_p{}^2{}_{-1}}\right); \quad k_p' = \frac{\partial\,\kappa_{p,n}}{\partial\,\kappa_p}\left(\frac{\kappa_{p,n}}{\mu_p{}^2{}_{-1}} - \frac{\kappa_{p+1,n}}{\mu_p{}^2}\right);$$

$$x_p = \mu_{p-1}\,\mathrm{R}_p\,\frac{\partial\,\kappa_{1,p}}{\partial\,\kappa_p} - \kappa_{1,p-1} = \mu_p\,\mathrm{R}_p\,\frac{\partial\,\kappa_{1,p}}{\partial\,\kappa_p} - \kappa_{1,p};$$

$$x_p' = \mu_{p-1}\,\mathrm{R}_p\,\frac{\partial\,\kappa_{p,n}}{\partial\,\kappa_p} + \kappa_{p,n} = \mu_p\,\mathrm{R}_p\,\frac{\delta\,\kappa_{p,n}}{\partial\,\kappa_p} + \kappa_{p+1,n}.$$

Then x_p and x_p' evidently depend on the position and curvature of the p^{th} surface only, and not on the refractive index of the following medium. Advantage may be taken of this to avoid multiplication by μ_{p-1} or μ_p, by splitting up each cemented surface into two glass air surfaces. Refraction into the air is then neglected in calculating k_p and k_p' while refraction into glass is neglected in calculating x_p and x_p'. The formulæ for the first order aberration coefficients are

$$\mathrm{A} = \sum_{p=1}^{n} k_p\,x_p{}^2; \quad \mathrm{A}' = \sum_{1}^{n} k_p'\,x_p'^2;$$

$$\mathrm{B} = \sum_{1}^{n} k_p\,x_p\,x_p'; \quad \mathrm{B}' = \sum_{1}^{n} k_p'\,x_p\,x_p';$$

$$\mathrm{C} = \sum_{1}^{n} k_p\,x_p'^2 = \sum_{1}^{n} k_p'\,x_p{}^2; \quad \tilde{\omega} = \sum_{1}^{n} \frac{\kappa_p}{\mu_{p-1}\cdot\mu_p}$$

The calculations are obviously very easy.

In the case of the telescope objective only A and B are required. $\mathrm{A}=0$ is the condition of freedom from first order spherical aberration: $\mathrm{A}-\mathrm{B}=0$ is the condition of freedom from first order coma: $\mathrm{B}=0$ is the sine condition to the same order. The only other quantity which is actually required for telescope objectives is the coefficient giving the second order spherical aberration. Let A_p and B_p be written for $k_p\,x_p^2$ and $k_p\,x_p\,x_p^1$ respectively, and $\mathrm{A}_{\lambda,\lambda'}$ and $\mathrm{B}_{\lambda,\lambda'}$ for $\sum_{p=\lambda}^{\lambda'} \mathrm{A}_p$ and $\sum_{\lambda}^{\lambda'} \mathrm{B}_p$. It must be noted that $\mathrm{A}_p, \mathrm{B}_p, \mathrm{A}_{\lambda,\lambda'}, \mathrm{B}_{\lambda,\lambda}{}'$ are not the A's and B's for the corresponding portions of the system: thus $\mathrm{A}_{\lambda,\lambda}{}'$ does not measure the spherical aberration for an object at infinity for the system $\lambda, \lambda_{+1}, \ldots \lambda'$. They are merely the contributions to A and B from the surfaces concerned when the calculations are made in accordance with the formulæ given above. The second order spherical aberration coefficient D is given by the formula

$$D = \sum_{p=1}^{n} A_p \left\{ 2\left(B_{1,p} - B_{p,n}\right) + \left(R_p \frac{\partial \kappa_{1,p}}{\partial \kappa_p}\right)^2 - \left(\frac{\kappa_{1,p}}{\mu_p^2} + \frac{\kappa_{1,-1p}}{\mu_{p\cdot 1}^2}\right) r_p \right\}$$

All the quantities appearing in this formula have already been found; only multiplication is involved.

If now a ray is incident parallel to the axis at a height h from the axis it will meet the paraxial image plane at a distance

$$\tfrac{1}{2} A h^3 + \tfrac{3}{8} D h^5$$

from the axis, and the longitudinal spherical aberration is

$$\tfrac{1}{2} A h^2 + \tfrac{1}{8} \left\{ 3D + 2A(B + 1) \right\} h^4.$$

In any actual system of course there are third and higher order aberrations. Comparison of the results obtained from trigonometrical calculations with the approximation given by the formula taking the first two aberrations into account will show how far such formulæ are to be relied on and what allowances in large aperture lenses are to be made for the neglected terms. My own experience with calculations made by this method is that when intelligently used they enable a great deal of time to be saved and can be relied on to give correct results. It is, however, essential to obtain a knowledge of the cases in which high order aberrations are likely to be of importance in comparison with those of the first and second orders. Generally speaking this is most likely to occur when (*a*) the curvature of the refracting surface is very great, (*b*) the difference in the refractive indices on the two sides of a refracting surface is small.

As a further illustration of the calculation of a lens by this method the objective used as an example by Mr. Everitt may be taken. Here the relative aperture is less than $f/10$ and the curvatures given by the first approximation neglecting thicknesses will be sufficiently accurate. From the figures given for the C and F lines the refractive indices for the D line may be interpolated, giving

	C	D	F	ν
Crown ...	1·57425	1·57725	1·58439	56·928
Flint ...	1·61105	1·61585	1·62779	36·789

the calculation of the ν to this number of figures simply being an expedient to get K and K′ to satisfy the colour condition as well as possible. For the D line

$K = 2{\cdot}826754$ $\quad R = 4{\cdot}896932$

$K' = 1{\cdot}826754$ $\quad R' = -2{\cdot}966232$

where the number of figures retained is kept to show how the final results compare with those obtained by trigonometrical methods. For the other lines

$$\begin{array}{rr@{\qquad\qquad}rr} K_C = & 2{\cdot}812063 & K_F = & 2{\cdot}661718 \\ K'_C = & -\,1{\cdot}812516 & K'_F = & -\,1{\cdot}862171 \\ \hline & 0{\cdot}999547 & & 0{\cdot}999547 \end{array}$$

showing that C and F focus together.

Returning to the D line

$$\varpi = 1{\cdot}792204 - 1{\cdot}130522 = 0{\cdot}661682$$

$$R + R' - \varpi = 1{\cdot}269018$$

and therefore

$$C - B = 6{\cdot}553434 + 1{\cdot}661682\,r$$

Again

$$\begin{array}{rr} KR^2 = & 67{\cdot}785400 \\ K'R'^2 = & -\,16{\cdot}072754 \\ 2KK'(R + R' - \varpi) = & -\,13{\cdot}105870 \\ \hline & 38{\cdot}606776 \end{array}$$

and thus

$$C = 38{\cdot}606776 + 19{\cdot}113254\,r + 2{\cdot}323364\,r^2$$

and

$$A = 26{\cdot}499908 + 15{\cdot}789890\,r + 2{\cdot}323364\,r^2$$

The solutions of $A = 0$ are

$$r = -\,3{\cdot}398066 \pm 0{\cdot}375522$$

$$\text{or}\quad r = -\,3{\cdot}773588 \quad\text{or}\quad -\,3{\cdot}022544$$

Taking the solution with the shallower cemented surface the curvatures are

$$\begin{array}{r} 1{\cdot}874388 \\ -\,3{\cdot}022544 \\ -\,0{\cdot}056312 \end{array}$$

or the radii for a focal length of 350 mm. are

186·728 mm. for the crown surface,
− 115·796 mm. for the cemented surface,
and − 6215·372 mm. for the flint surface.

It is quite sufficient, as has been said, to take

186·7 mm.
− 115·8 mm.
− 6215 mm.

as the final solution. It will however be interesting to take the exact radii that have been found and see what errors remain with thicknesses of 4 mm. and 2 mm. for the crown and flint components. Using Mr. Everitt's notation, the results for paraxial rays are :—

$A^{C} = 346·056$ $F^{C} = 349·990$
$A^{D} = 345·878$ $F^{D} = 349·816$
$A^{F} = 345·985$ $F^{F} = 349·935$

The distance between the centres of curvature of the first and second surfaces is $-298·524$ mm. and between the second and third $-6097·576$ mm.

If ϕ and ϕ' are the angles of incidence and refraction, ψ the angle between the ray and the axis, and *h* the distance between the ray and the centre of curvature, the complete detailed calculation of edge rays through the system is as follows, a calculating machine and a table of natural sines to seven figures being used:

C LINE.

	Angle.		Sine.	*h* and Δh.
	Deg.	Min.		
ψ_0	0	0	0·0	
ϕ_1	4	36·4552	0·0803309	15
ϕ_1'	2	55·4978	0·0510281	9·52835
ψ_1	1	40·9574	0·0293631	8·76559
ϕ_2	-9	5·3923	$-0·1579835$	18·29394
ϕ_2'	-8	52·8326	$-0·1543749$	17·87607
ψ_2	1	28·3977	0·0257111	156·77537
ϕ_3	-1	36·6131	$-0·0280999$	174·65144
ϕ_3'	-2	35·6813	$-0·0452704$	281·37220
ψ_3	2	27·4659	0·0428829	

6561·408
6215·372

$A^{C} = 346·036$ $F^{C} = 349·790$

D LINE.

	Angle.		Sine.	*h* and Δh.
	Deg.	Min.		
ϕ_1'	2	55·1635	0·0509310	9·51022
ψ_1	1	41·2917	0·0294603	8·79461
ϕ_2	-9	5·7198	$-0·1580776$	18·30483
ϕ_2'	-8	52·5769	$-0·1543014$	17·86756
ψ_2	1	28·1488	0·0256387	156·33391
ϕ_3	-1	36·3641	$-0·0280275$	174·20147
ϕ_3'	-2	35·7429	$-0·0452883$	281·48345
ψ_3	2	27·5276	0·0429009	

6561·248
6215·372

$A^{D} = 345·876$ $F^{D} = 349·643$

F LINE.

	Angle.		Sine.	h and Δh.
	Deg.	Min.		
ϕ_1'	2	54·3737	0·0507015	9·46737
ψ_1	1	42·0815	0·0296899	8·86315
ϕ_2	− 9	6·4920	− 0·1582994	18·33052
ϕ_2'	− 8	51·8024	− 0·1540788	17·84179
ψ_2	1	27·3919	0·0254186	154·99183
ϕ_3	− 1	35·6072	− 0·0278074	172·83362
ϕ_3'	− 2	35·6617	− 0·0452647	281·33684
ψ_3	2	27·4464	0·0428773	

6561·440
6215·372

$A^F = 346·068$ $F^F = 349·835$

It will be seen that the system of calculation is appreciably different from that employed by Mr. Everitt. It has the merit of being very quick, a ray taking 15 minutes or less to be traced through three surfaces to seven figure accuracy, but is not so well adapted to logarithmic computation.

The final results are thus as follows :—

	PARAXIAL RAYS	MARGINAL RAYS
A^C	346·056	346·036
A^D	345·878	345·876
A^F	345·985	346·068
F^C	349·990	349·790
F^D	349·816	349·643
F^F	349·935	349·835

or separating out aberrations as in Mr. Everitt's scheme

CHROMATIC ABERRATION

Axis	0·07 mm. uncompensated
Edge	0·03 mm. overcompensated

SPHERICAL ABERRATION

C	0·02 mm. uncompensated
F	0·08 mm. overcompensated

CHROMATIC ABERRATION OF THE FOCAL LENGTH

Axis	$0·05_5$ mm. uncompensated
Edge	$0·04_5$ mm. overcompensated

ERROR AGAINST THE SINE CONDITION

C	0·20 mm. uncompensated
F	0·10 mm. uncompensated.

showing that the objective is satisfactorily corrected.

Instead of using the error against the sine condition it is better to determine the comatic displacement for secondary rays. Let the image plane be taken through the paraxial focus for sodium light. The length of the refracted ray intercepted between the incident ray and this plane is

$$F - (A - 345{\cdot}878) \sec \psi_3$$

where the F and A refer to the refracted marginal ray. The results are —sec ψ being unity to the accuracy required—

C	349·632
D	349·645
F	349·645

The value for no coma in this image plane would be 349·816, the value of $F^{\prime\prime}$ for the paraxial ray. The minimum change of magnification for edge rays due to coma is thus about 1 part in 2,000. The maximum change will not be more than between 3 and 5 times as great; 1 part in 500 will be a fairly safe upper limit.

If the spherical aberration, coma, curvatures of primary and secondary focal surfaces, distortion, and variation of distortion are denoted by δ_1, δ_2, $3\delta_3$ and $\delta_3 + \frac{2}{3}\tilde{\omega}(S-m)^2$, δ_4, and δ_5 respectively for an object magnification m and stop magnification S, these aberrations are measured by multiplying the quantities at the heads of the following columns by the factors written below and adding together the products.

	A	$-\left(B+\frac{1}{4}\right)$	$\left(c+\frac{\tilde{\omega}}{8}\right)$	$-\left(B'+\frac{1}{4}\right)$	A'	$\frac{1}{4}\left(S-m\right)$
δ_1	1	$4m$	$6m^2$	$4m^3$	m^4	0
δ_2	1	$3m + S$	$3m(m+S)$	$m^2(m+3S)$	m^3S	$1 - m^2$
δ_3	1	$2(m+S)$	$m^2 + 4mS + S^2$	$2mS(m+S)$	m^2S^2	$2(1 - mS)$
δ_4	1	$m + 3S$	$3S(m+S)$	$S^2(3m+S)$	mS^3	$3(1 - S^2)$
δ_5	1	$4S$	$6S^2$	$4S^3$	S^4	0

It will be noted that there is a definite connexion between the aberrations of the object and those of the stop taken in the reverse order, the factors in the last column representing inconsistencies between the two. All the essential connexions between first order aberrations are implied in the above coefficients. When all the aberrations vanish

$$\frac{A-m}{m^3} = \frac{B}{m^2} = \frac{C}{m} = \frac{\tilde{\omega}}{0} = B' = A'm - 1 = -\frac{S^3 - m}{(S-m)^3}$$

APPENDIX.

Since the above was written Mr. Everitt has communicated the actual radii of his solution. It appears from a recalculation of his objective that the number of figures he has used allows an error of about 5 in the second decimal place of his focal lengths and intersection distances. When more figures are retained the values found by Mr. Trump and myself using different methods are as follows :—

	PARAXIAL RAYS.	MARGINAL RAYS
A^C	346·157	346·155
A^F	346·097	346·214
F^C	349·963	349·827
F^F	349·918	349·883

and for the aberrations :

CHROMATIC ABERRATION.

Axis	·06 mm	uncompensated
Edge	·06 mm	overcompensated

SPHERICAL ABERRATION.

C	·00 mm	
F	·12 mm	overcompensated

CHROMATIC ABERRATION OF THE FOCAL LENGTH.

Axis	$\cdot 04_5$	uncompensated
Edge	$\cdot 05_5$	overcompensated

ERROR AGAINST THE SINE CONDITION.

C	$\cdot 13_5$	uncompensated
F	$\cdot 03_5$	uncompensated.

These figures show that the outstanding aberrations are less than Mr. Everitt's values and are very close to those of my second example. There is a slight difference between the two objectives in the type of colour correction secured, and the shorter radius of the cemented surface in Mr. Everitt's construction leads to an objective with slightly less outstanding coma. The one may be derived from the other very approximately by the addition (algebraic) of $\frac{1}{6}$ dioptre of curvature to each surface of the system. The addition of any intermediate amount would probably lead to an objective as good as was required. The possibility of making such common curvature additions within suitable limits constitutes one of the chief advantages to be gained by cataloguing tools in terms of their curvatures rather than of their radii.

T. S.

Calculation of Telescope Objectives free from Chromatic and Spherical Aberrations.

By S. D. CHALMERS, M.A.

THE method used in the calculations is to obtain approximate expressions for the various aberrations, by neglecting the effects of thickness and the higher order aberrations. From these expressions approximate curves are deduced, these curves are checked by trigonometrical calculations and the various errors estimated.

The approximate expressions are now used to calculate the changes in the curves to correct these errors. If necessary the curves are again checked by trigonometrical methods.

The trigonometrical calculations follow very closely the methods described by Steinheil and Voit, except in a few minor points to which attention will be called. The method is illustrated in the appended account of the calculation of a doublet free from Spherical Aberration, Chromatic Aberration and Coma.

The chromatic aberration of the two lens system is expressed by

$$\frac{F_1 + F_3}{\nu_2} + \frac{F_5 + F_7}{\nu_6}$$

where F_1, F_3, F_5, and F_7 are the powers of the surfaces and ν_2 and ν_6 are the dispersion coefficients of the two glasses used.

The lateral spherical aberration δx is given by $\delta x = \frac{1}{2} f e A_8^3 M_6^3$ where f is the focussing distance, A_8 the semi-aperture angle of the complete system, M_6 the linear magnification produced by the 2nd lens; e is a constant depending on the construction of the system.

$$e = -\frac{1}{n_2(n_2-1)^2}\left\{m_3^2(n_2+2)(1-M_2) - m_3 n_2(2n_2+1)(1-M_2^2) + n_2^3(1-M_2^3)\right\}$$
$$+\frac{1}{n_6(n_6-1)^2}\left\{\frac{1}{m_5^2}(n_6+2)\left(1-\frac{1}{M_6}\right) - \frac{1}{m_5}n_6(2n_6+1)\left(1-\frac{1}{M_6^2}\right) + n_6^3\left(1-\frac{1}{M_6^3}\right)\right\}$$

where n_2 and n_6 are the refractive indices of the two glasses, m_3 and m the linear magnifications due to the 2nd and 3rd surfaces, M_2 and M_6 the linear magnifications due to the lenses.

For convenience $\frac{1}{M_6}$ will be written P_6 or P

$\frac{1}{m_5}$,, ,, p_5 or p

and m_3 ,, ,, m

The condition for the correction of coma is $i = 0$ where

$$i = \frac{1}{(n_2 - 1)\, n_2} \{m_3 (n_2 + 1)(1 - M_2) - n_2^2 (1 - M_2^2)\}$$

$$- \frac{1}{(n_6 - 1)\, n_6} \{p_5 (n_6 + 1)(1 - P_6) - n_6^2 (1 - P_6^2)\}$$

The condition for achromatism becomes

$$\frac{1 - M_2}{\nu_2} = \frac{1 - P_6}{\nu_6}$$

A practical calculation for the case $M_2 = 0$ (*i.e.* the object at infinity) is illustrated.

In this case $P = \frac{\nu_2 - \nu_6}{\nu_2}$

For the glasses $n_2 = 1{\cdot}5168$ $\nu_2 = 60{\cdot}6$

$n_6 = 1{\cdot}6118$ $\nu_6 = 36{\cdot}9$

$P = {\cdot}3911$, and with this value of P

$$e = -\{8{\cdot}681\, m^2 - 15{\cdot}103\, m + 8{\cdot}614\} + \{3{\cdot}645\, p^2 - 9{\cdot}558\, p + 6{\cdot}525\}$$

and the condition for no coma is

$$3{\cdot}211\, m - 2{\cdot}935 - (1{\cdot}613\, p - 2{\cdot}231) = 0$$

Eliminating m we find $p = {\cdot}2265$ or $2{\cdot}4436$; the former is the more suitable value and leads to

$$m = {\cdot}33302$$

With these values of P, m and p the powers of the surfaces for a focal length of 1 metre are evaluated; suitable thicknesses 10 mm. and 6 mm. are chosen; the powers of the surfaces as calculated are taken as the equivalent or effective powers of the surface as used and the corresponding radii of curvature calculated and found to be

$r_1 = 606{\cdot}93$ mm.

$r_3 = -\ 301{\cdot}34$ mm.

$r_5 = -\ 307{\cdot}60$ mm.

$r_7 = -1442{\cdot}15$ mm.

With these curves and a semi aperture of $h = 50$ mm. the focussing distances were calculated.

D line	$h = 50$	$v = 992{\cdot}191$
	$h = 0$	$v = 992{\cdot}208$
violet line	$h = 50$	$v = 992{\cdot}602$
$n_V = n_D + n_F - n_C$	$h = 0$	$v = 992{\cdot}128$

Following Steinheil* the ordinary trigonometrical formulæ are modified by taking circular measures instead of sines for the paraxial ray throughout, but it will be found convenient to multiply all these angles by a constant, so that the logarithms are comparable with those for the actual ray $h = 50$.

The results of the calculation† show that the spherical aberration is substantially corrected, but the chromatic difference of spherical aberration should be taken into account in determining the best chromatic correction.

In the actual example chosen it was decided to choose the smallest confusion circle for each colour and to bring these to a common focus.

This required that the chromatic aberration should be modified by ·288 mm., and we have

δR_7 the change in the last curvature is given by

$$- \delta R_7 (n_F - n_C) = \frac{\cdot 288}{v^2}$$

or

$$\delta r_7 = \frac{r_7{}^2}{v^2} \times \frac{\cdot 288}{\cdot 01658} = 36{\cdot}705$$

and the new $r_5 = - 1442{\cdot}15 + 36{\cdot}70 = - 1405{\cdot}445$

With this value of r_7 the calculations were repeated and gave

D line	$h = 50$	981·267
	$h = 0$	981·418
violet	$h = 50$	981·352
	$h = 0$	981·054

It is now required to correct the spherical aberration which is $\delta v = - \cdot 151$

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[*] It is, however, often preferable to use other methods of calculating the ideal ray and the chromatic correction for the ideal ray.

[†] The results of this calculation are quite typical for ordinary objectives.

To correct we must alter the form of the last lens. We have

$$\tfrac{1}{2} f \,\delta e\, A^3 \frac{1}{P_6^3} = A\,(-\,\delta v)$$

i.e.

$$\delta e = \frac{-\,\delta v\, P_6^3}{\tfrac{1}{2} f A^2} = \frac{-\,\delta v\, P_6^3}{\tfrac{1}{2} h\, A}$$

$$2\,\delta p\,(3{\cdot}645\,p - 4{\cdot}779) = \frac{{\cdot}151}{\tfrac{1}{2} \times 50 \times {\cdot}05055}$$

$$\delta p = \frac{{\cdot}151 \times P_6^3}{50 \times {\cdot}05055 \times \{(3{\cdot}645\,({\cdot}22649) - 4{\cdot}779)\}}$$

$$F_5 = -\,(1-p)\,F_{1,3}$$

$$l_5\, R_5\,(n_6 - 1) = -\,(1-p)\,F_{1,3}$$

$$\frac{\delta R_5}{R_5} = \frac{-\,\delta p}{1-p}$$

$$\frac{-\delta R_5}{R_5} = \frac{\delta r_5}{r_5} = \frac{\delta p}{1-p} = \frac{{\cdot}151 \times ({\cdot}3911)^3}{\left\{\underset{(1-p)}{{\cdot}77351}\,(\underset{(h)}{50} \times \underset{(A_8)}{{\cdot}05055})\right\}\left\{(3{\cdot}645 \times \underset{(p)}{{\cdot}22649}) - 4{\cdot}779\right\}}$$

$$\delta R_7 = \delta R_5$$

$$\frac{\delta r_5}{r_5} \times r_5 = \delta r_5 \qquad \delta r_5 + r_5 = \text{new } r_5$$

$$\frac{-\,\delta R_5}{R_5} \times -\,R_5 = +\,\delta R_5$$

$\delta R_7 = \delta R_5$ and new $R_7 = R_7 + \delta R_7$ (using corrected R_7)

This gives $r_5 = -\,307{\cdot}240$ and $r_7 = -\,1397{\cdot}98$

The re-calculation for the last two surfaces gives

			Circle of least confusion.
D line	$h = 50$	981·449	981·445
	$h = 0$	981·429	
violet	$h = 50$	981·554	981·417
	$h = 0$	981·037	

The accuracy of the calculation does not permit of the 2nd decimal figures being accurately obtained, and where this is necessary, as in theoretical investigations, it is desirable to carry the calculation to a further place of decimals in the last surface only, or to use a method similar to that described by Steinheil for surfaces that are nearly plane.

For the case of cemented lenses the actual calculations are somewhat simplified if we substitute

$$C_2 \text{ for } 1 - M_2 \text{ and } C_6 \text{ for } 1 - \frac{1}{M_6}$$

$$B_2 \text{ for } \frac{1 - m_3}{n_2 - 1} \text{ and } B_6 \text{ for } \frac{1 - {}_1/m_5}{n_6 - 1}$$

We then have

$$e = -\left\{\frac{2 + n_2}{n_2} B_2{}^2 C_2 + 4\frac{n_2 + 1}{n_2} B_2 C_2 - \frac{2n_2 + 1}{n_2 - 1} B_2 C_2{}^2 \right.$$

$$\left. + \frac{3n_2 + 2}{n_2} C_2 - \frac{3n_2 + 1}{n_2 - 1} C_2{}^2 + \left(\frac{n_2}{n_2 - 1}\right)^2 C_2{}^3\right\}$$

+ similar terms for the second lens.

If the object be at infinity $C_2 = 1$ and the condition for achromatism is

$$C_6 = \frac{\nu_6}{\nu_2}$$

The condition for a contact is now $B_2 = B_6$ and if this substitution be made the expression for $e = 0$ leads to a quadratic which can be solved for B_2.

As before, the change in e corresponding to a given change in B_2 can be found by differentiation.

In the above it has been assumed that the lens is to be perfectly corrected in itself; the procedure is only slightly varied when it is necessary to compensate errors of other systems.

As regards the spherical aberration, any desired over or under-correction may be allowed for in the original equations by equating e to the value corresponding to the desired correction. This value of e is computed from the desired longitudinal aberration in the same way as δe is computed above. Thus a desired over-correction of ·2 mm. could be secured by making

$$e = \cdot 2\frac{h}{f}\Big/\tfrac{1}{2}f\left(\frac{h}{f_2}\right)^3$$

where h is the semi aperture, f the focal length of the whole lens, and f_2 that of the first lens. A similar procedure may also be adopted with regard to coma.

As regards chromatic aberration, it is preferable to express the desired over-correction as a fraction of the focal length, say $a.f$.

Then

$$\frac{F_2}{\nu_2} + \frac{F_6}{\nu_6} = -a\,(F_2 + F_6)$$

giving

$$\frac{F_2}{\nu_2} = -\frac{F_6}{\nu_6\dfrac{1 - a\nu_2}{1 - a\nu_6}}$$

as the condition from which $\frac{1}{M_6} = P$ can be found.

Whenever an objective of similar type has been calculated, the errors found by trigonometrical calculation may be used to find the most suitable values to assign to e and a in the original equations. In this way the trigonometrical work can be avoided except as a final check upon the whole work.

Interpretation of Results.

The spherical aberration is given by the trigonometrical calculation as a longitudinal error, *i.e.* an error of focussing position.

For telescope objectives (with spherical surfaces and homogenous glasses) the lateral spherical aberration may be assumed to be sufficiently accurately represented by

$$\delta x = -\tfrac{1}{2} f (e_1 A^3 + e_2 A^5)$$

where e_1 and e_2 are two aberrational coefficients which could be determined from the results of two suitable rays calculated through the system and the paraxial ray.

In the case of low magnification telescopes the performance will depend on the diameter of the image patch, and it will usually be possible to select the focus to reduce this diameter as much as possible.

It is customary to make the aberration at the edge zero, giving a maximum value which can be computed from the value $A = \sqrt{\cdot 6}\, A$ max. The best choice of focus reduces this error to about ·45 of its value. It is, however, preferable to correct for the aperture ·95 of the total.

A diameter of the image patch is permissible, such that it corresponds to an angular value at the eye of from $\tfrac{1}{2}$ to 1 minute ; thus the calculated intermediate aberration might correspond to $\tfrac{1}{2}$ to 1 minute in radius.

The permissible longitudinal error, after allowing for *e.p.* correction is thus

$$\tfrac{1}{2}\frac{f\,e.p.}{3400} \times \frac{f\,o.g.}{h\,o.g.}$$

to twice this value, according to the definition required.

As an example, if $f/h = \frac{1}{10}$ $f = 20$ mm. ; this becomes ·03 mm. to ·06 mm.*.

When, however, the magnification is high, other considerations come into play, and the value of the definition is best determined by evaluating the brightness in the centre of the image. In effect this is secured by arranging that the phase differences in the light reaching the axial image should not differ by more than $\tfrac{1}{4}$ phase, *i.e.* $\tfrac{1}{8}$ from the mean.

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* ·05 mm. would in this case correspond to an error in focus due to an observer requiring a correction of $+\tfrac{1}{8}$ D.

A simple method of evaluating these phase differences is to plot the lateral aberrations against the aperture.

The area of the curve up to the point considered, divided by the focal length, will represent the wave length difference.

The effect of changing the focus is shown by taking a line through the focus oblique to the focal plane as the base of the figure instead of the focal plane line.

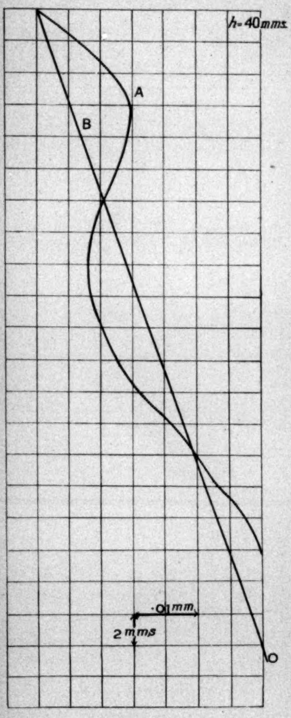

Fig. I.

Objective 80 mm. diameter. Focal length 1 metre, re-calculated from Hartmann's measurements.
A lateral aberration curve.
B zero line for modified focus.

In Fig. I. is shown the lateral aberration curve of a photographic objective of 80 mm. free aperture and 1 m. focal length. The curve is drawn for $\lambda = 540\,\mu\,\mu$ from the longitudinal aberrations as measured by Hartmann* who states that this objective had been specially figured to avoid zonal aberration.

It is evident from the shape of the curve that the lens was designed so that the edge ray came to the theoretical focus, but the outer zone has been modified by figuring which has increased its aberration. It is also evident that the best focus is not the theoretical focus, but is represented by the line O A which is equivalent to a focus $\frac{35}{1000} \times \frac{1000}{40} = \cdot 88$ mm. nearer the objective. This corresponds very closely with Hartmann's statement that for white light the best focus is $\cdot 65 \pm \cdot 15$ mm. nearer the objective than the zero plane from which our measurements are recorded. It will be noticed that the line O A is drawn to cut off from the curve areas in the ratios 1 : 2 : 2 thus making the path difference as small as possible and its greatest value $= 36\,\mu\,\mu = \frac{\lambda}{15}$ approx., which is well within Rayleigh's standard.

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* Zeitschrift für Instrumentenkunde, p. 28, Jan. 1904.

Fig. II*a*. shows the curve of the camera objective of a spectrograph $f = 356$ mm. $2\,h = 30$ mm.; this objective had been specially refigured by Steinheil to correct a certain amount of existing zonal aberration; the figure shows that this had been accomplished by considerably increasing the actual aberration, thus permitting of a choice of focus considerably removed from the theoretical focus.

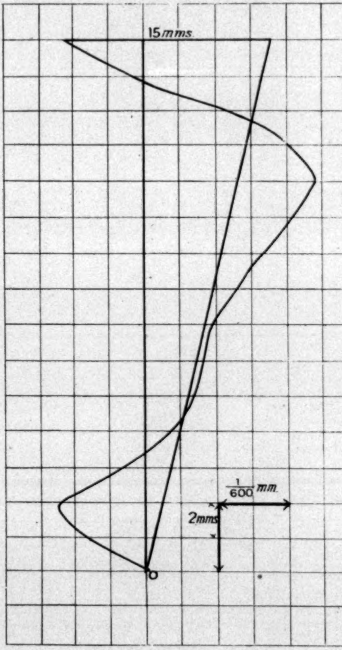

Fig. II. *a*

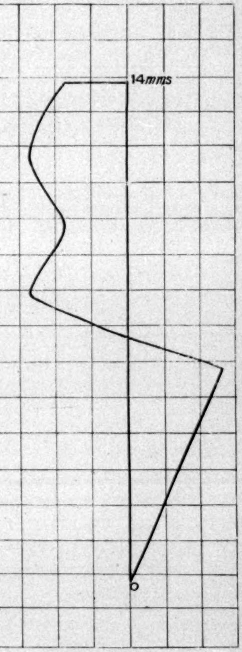

Fig. II. *b*.

In Fig. III. is indicated the result of refiguring an objective $f = 305$ mm. $2\,h = 30$ mm., a triple cemented by Steinheil used as the collimator of the same spectrograph. The cementing medium had caused an appreciable change in the focus and definition and it was thought desirable to refigure;

the original form gave definition just outside the limits of Rayleigh's criterion, but in actual use the total path difference might have been twice

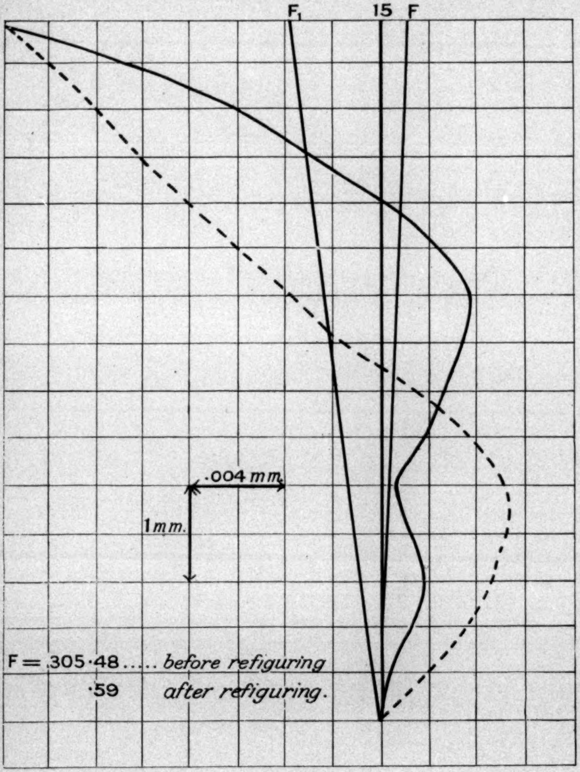

Fig. III.

this amount. The definition for the standard colour was improved by refiguring, which gave a path difference of $\frac{1}{10}\lambda$.

A lens (Fig. II. *b.*) similar to the camera objective gave a maximum path difference $\frac{\lambda}{25}$. For small objectives, therefore, it seems that the definition requires a path difference of less than $\frac{\lambda}{10}$ from the mean (approx.), and this is generally obtained by local figuring of the objective. Fig. IV. indicates the lateral aberration curve for the objective $f = 1$ m. approx. $2\,h = 100$ mm. calculated above. If the focus be considered to be at the theoretical focus there is a lateral aberration of ·0036 and a path difference of $\frac{\lambda}{10}$; choosing the best focus this reduced to $\frac{\lambda}{40}$. But it is evident that the correction has not been chosen to give quite the best possible result; Conrady* has suggested that the path difference should be zero for the outside ray, but this does not take account of the possibility of choosing a focus other than the theoretical focus.

With sufficient approximation the effect of an alteration of the correction can be indicated by measuring the aberrations from a curve O D given by $x = ey^3$; thus, instead of the lateral aberration curve A O, we would get the curve whose abscissæ are represented by the differences between the abscissæ of O A and O D; again, the effect of a change of focus can be represented by measuring from a straight line O F.

In this way it will always be possible to choose O D and O F so that the maximum value of the path difference is as small as possible; this will generally be when the areas are as 1 : 2 : 2 nearly. In the figure the lateral aberration at the edge is ·0035, *i.e.* ·07 longitudinal under-corrected, while the best focus is ·09 from central, the greatest path difference being $\frac{\lambda}{40}$. The best result is obtained by correcting the lens for an aperture ·96 of the full aperture.

The measurements given above are purposely taken from smaller objectives, because the results of figuring are seen as definite alterations of parts of the curve. It is interesting, however, to take the curve of a larger refractor, where the difference in path will almost certainly, for part of the aperture, exceed $\frac{1}{4}\,\lambda$. The choice of best focus is now complicated by the fact that the smallest area is not necessarily the most favourable when the path difference exceeds $\frac{\lambda}{4}$. For this purpose it is necessary to determine, as Strehl has done, the intensity curve and choose the greatest intensity, but the focus can in some cases be easily obtained.

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* Royal Astronomical Soc. 1905.

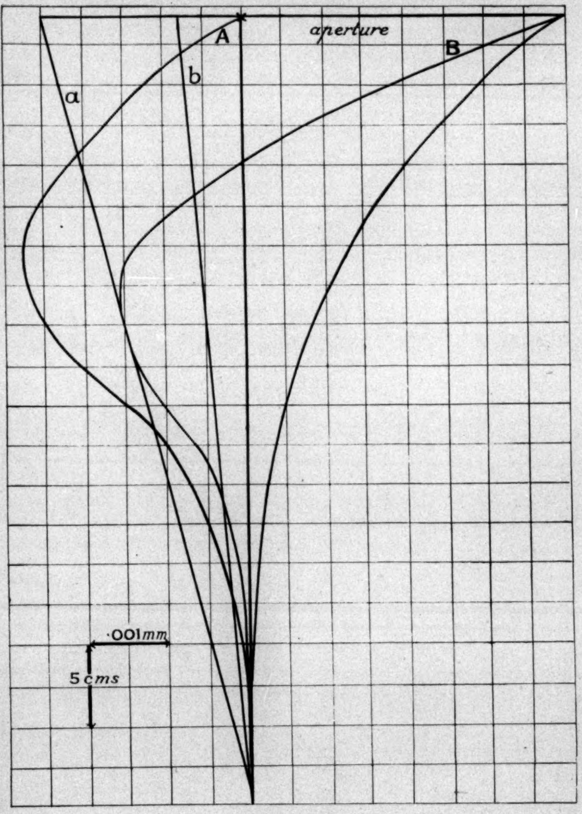

Fig. IV.

All calculated curves.

A curve aberration for edge ray corrected.
a ,, best focus reference line.
B curve Conrady's condition.
b ,, Zero for this condition

Fig. V. represents the Potsdam Refractor $f = 3400$ $2h = 320$ mm., and it is desirable, as far as possible, to choose the focus so that the points, at which the curve is steepest, correspond as nearly as possible to odd multiples of $\frac{1}{2}\lambda$ differences of path from the mean.

The Sine Condition.

The departure from the sine condition furnishes a measure of the coma of an optical system, but the relation is only simple in the case where the spherical aberration is supposed corrected and the coma is regarded as an aberration of the lowest order only.

For the present purpose the coma will be defined as the defect which varies as the 1st power of the angular field and depends on the 2nd and higher powers of the aperture.

The coma of the 1st order produces an error in each ray of which the co-ordinates as a lateral aberration are

$$-\tfrac{1}{2} f i \sin^2 A (\cos 2j + 2) \tan \theta$$
$$-\tfrac{1}{2} f i \sin^2 A \sin 2j \tan \theta$$

where f is the focal length, i is a coefficient, A is the aperture angle, θ the angular field, and j the polar co-ordinate of the point on the aperture considered.

As the point on the aperture travels round a circle, the image point travels round a circle at twice the rate, the radius of this circle is $\frac{1}{2} f i \sin^2 A \tan \theta$, while its centre is at twice this distance from the theoretical image point.

Now when $j = 90^\circ$ (*i.e.* the ray proceeds in the sagittal plane) the image point is at a distance of $\frac{1}{2} f i \sin^2 A \tan \theta$ from the theoretical image point.

I have shown* that a ray proceeding from a point just off the axis to such a point on the aperture will cross the plane containing the axis and the object point at a definite distance from the axis depending on quantities which can be deduced from the axial ray only.

The result is that this distance d is equal to

$$\frac{n_0 \sin A_0}{n \sin A} x_0$$

where x_0 is the object size, A_0 and A the angles the axial ray for the same aperture makes with the axis, and this leads to

$$\frac{n_0 x_0 \sin A_0}{n_2 x_2 \sin A} - 1 = \frac{\delta v}{v} + \frac{\delta x_2}{x_2}$$

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* Proc. Phys. Soc., Vol. XXII, p. 1.

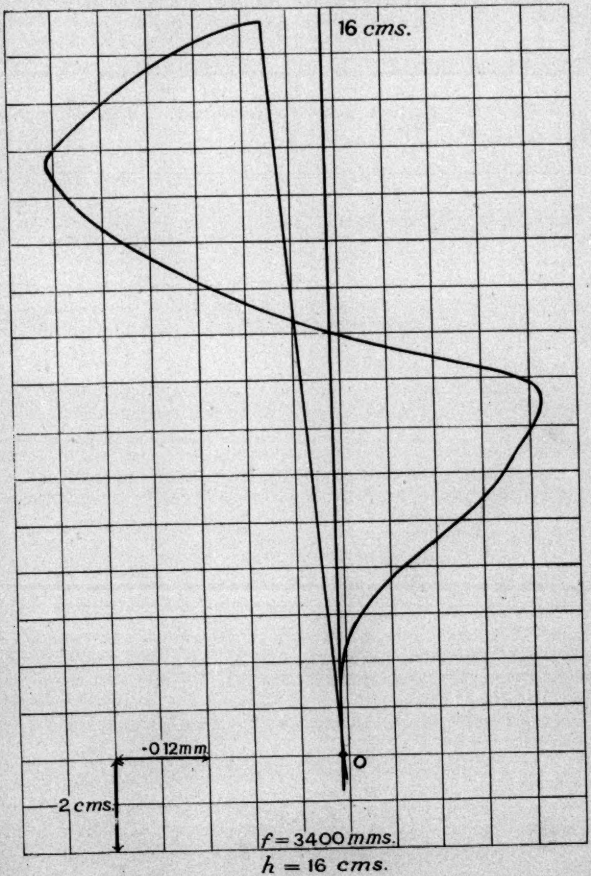

Fig. V.

where v is the distance from the stop plane to the image plane, δv the longitudinal spherical aberration and δx_2 the comatic aberration for this ray.

$\frac{\delta x_2}{x_2}$ can thus be deduced from the trigonometrical result. To obtain the maximum coma effect it is necessary to deduce $\frac{\delta x}{x}$ for two rays and express the result as

$$\frac{\delta x_2}{x_2} = a_1 A^2 + a_2 A^4$$

the maximum value of the coma is then

$$3 a_1 A^2 + 5 a_2 A^4$$

and it is evident that the sine condition may be satisfied for extreme apertures, leaving a large residual defect for a smaller aperture.

In this case the satisfaction of the sine relation is no longer an indication of the absence of coma for this zone, but merely that the double circle has one point at the theoretical image.

These considerations are not of great importance in ordinary telescope systems, but become important as aperture ratios are increased.

For complete telescope systems, the stop position being the same for *o.g.* and *e.p.*, the effects of spherical aberration can be left out of account when the spherical aberration of the *o.g.* balances that of the *e.p.*

DISCUSSION.

MR. J. W. HASSELKUS writes :—

Mr. Everitt's paper applies only to the case of long-focus objectives for astronomical telescopes or collimators. I would like to explain how very differently the correction must be arranged for a short-focus objective, for instance, for a 6 power prismatic binocular.

The eye-piece of say about 20 mms. focus is first designed as well as possible consistent with all the conditions that have been imposed. These may have been—

(1) a limited choice of glass material,
(2) the maximum exit beam to be 5 mms.,
(3) the field of view to be 50 degrees,
(4) that no radius should be shorter than 13 mms.,
and (5) that two plane surfaces should be retained.

The two latter conditions are introduced purely for manufacturing reasons.

If we set out on this work with, say, Heavy Flint and Hard Crown for the eye lens and Baryta Light Flint for the field lens, we find that one of the solutions will show the following result—

The distances of intersection from the last surface for C, D and F are—

S'_C ... 8·17 for the central rays and $S'_C\,m$ ... 7·46 for the marginal rays.
S'_D ... 8·14 $S'_D\,m$ ... 7·44
S'_F ... 8·13 $S'_F\,m$ ... 7·43

The focal lengths for C, D and F for centre and margin respectively are—

F_C ... 22·17 and $F_C m$ ... 22·08
F_D ... 22·11 $F_D m$ ... 22·03
F_F ... 22·01 $F_F m$ ... 21·93

The result is shown graphically in Fig. *a*, the errors being greatly exaggerated. The wave-lengths are taken as ordinates and the errors as abscissæ.

It is evident from the figure

(1) that the chromatic correction is perfect.

(2) that a fair amount of spherical aberration remains for a 5 mms. beam, and

(3) not only that the red focus is considerably longer than the blue, but, in addition, the foci of the marginal rays are all shorter than those of the central rays. This means that the size of the image depends upon the wave-lengths and also upon the zone responsible for its production. This is about the best we can do to suit the conditions imposed. Give the designer greater freedom, and a good deal of improvement can be effected.

I will now explain what would be the ideal correction an objective should have to combine with the eye-piece I have just dealt with. We will assume the objective to be 120 mms. focus and 30 mms. aperture and the whole combination to give a power of about 6. We require—

(1) absolute chromatic correction for C, D and F,

(2) spherical over-correction equal to the undercorrection present in the eye-piece,

(3) the chromatic difference of magnification to be six times larger than in the eye-piece, but in the same direction, *i.e.*, over-corrected for C and undercorrected for F, and

(4) the error against the sine condition to be six times larger than in the eye-piece and in the same direction, *i.e.*, under-corrected.

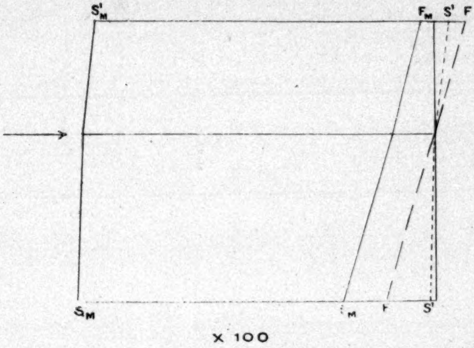

Fig. *a*.

The impossibility of arriving at such a correction can be seen at once from the following figures and the diagram (Fig. *b*)—

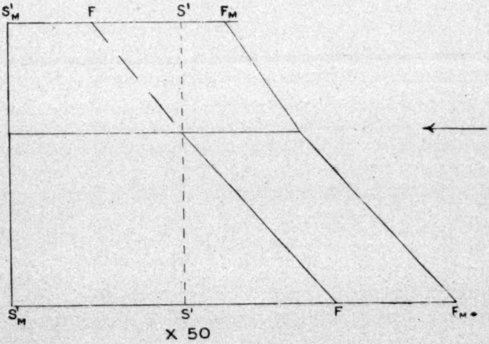

Fig. *b*.

S'_C	... 117·59	$S'_C m$	... 118·29	F_C	... 124·69	$F_C m$	... 124·15
S'_D	... 117·59	$S'_D m$	... 118·29	F_D	... 124·33	$F_D m$	... 123·85
S'_F	... 117·59	$S'_F m$	... 118·29	F_F	... 123·73	$F_F m$	... 123·25

The correction as existing in the objective mentioned by Mr. Everitt is shown as follows (see also Fig. *c*)—

S'_C	... 346·18	$S'_C m$	.. 346·13
S'_D	... 345·94	$S'_D m$	... 345·91
S'_F	... 346·14	$S'_F m$	... 346·21
F_C	... 350·00	$F_C m$	... 349·87
F_D	... 349·76	$F_D m$	... 349·65
F_F	... 349·96	$F_F m$	... 349·86

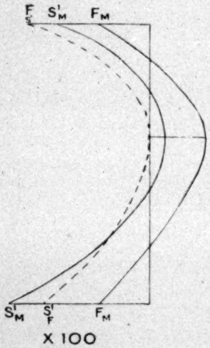

Fig. c.

It will be seen how far removed this state of correction is from the ideal one. However, we will see how near we can get to the correction we require.

The conditions to be fulfilled are
(1) achromatic correction for 3 colours over the whole aperture,
(2) a definite amount of spherical aberration,
(3) a definite amount of error against the sine condition, and
(4) a given focus.

The type of objective chosen is the cemented doublet, and, as a good number of conditions have to be fulfilled, we have to start with a special pair of glasses. Even then we cannot fulfil the condition of achromatism for 3 colours over the whole aperture, but it will be found that we can come fairly close to what we want.

The selected glasses are a Light Flint of about 1·59 refractive index and a Baryta Light Flint of 1·56 refractive index, ν being 41 and 55 respectively.

Fig. *d* shows the actual correction and in what respect it is different from the ideal. As a net result we have an optical combination showing good definition and freedom from colour except for a surplus of blue, as can be seen in the diagram.

I have purposely left out the prisms from the combination, as it would confuse the issue. In practice they do alter the correction slightly and must be taken into consideration. I have also not touched upon the question of roundness of field, astigmatism and coma.

The calculation shows the following results :—

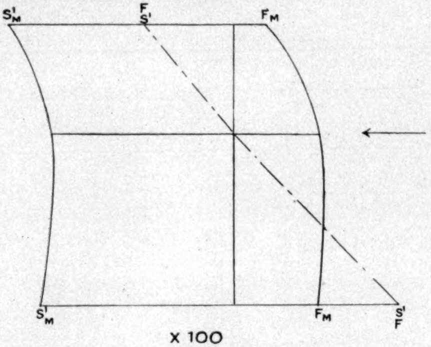

Fig. *d*.

S'_C	... 117·77	$S'_C\,m$	... 118·05	F_C	... 124·51	$F_C m$	... 124·26
S'_D	... 117·59	$S'_D\,m$	... 117·96	F_D	... 124·33	$F_D m$	... 124·16
S'_F	... 117·26	$S'_F\,m$	... 117·98	F_F	... 124·00	$F_F m$	... 124·16

In conclusion, I would say that the eye-piece shown is a type commonly employed, as very few other glasses are available, and the objective is a pre-war type. The correction required is difficult to get in a cemented doublet by combining Hard Crown or Borosilicate Crown with Heavy Flint.

MR. JAMES W. FRENCH writes :—

I hope Mr. Everitt will find an early opportunity of extending his very valuable paper on the design of objectives. A course of such papers, treated in the same concise and lucid manner, would be very helpful, especially to those younger members whose depressing task it is to search the present so-called treatises on optics for any information of real use to the practical computer. Of optical treatises there is no end. They are a by-product of our colleges and a monument to the regrettable divorce of the school and the workshop.

What text-book is there that tells us so plainly as does Mr. Everitt that first the approximate form is determined from existing experience of similar objectives, or, failing that, from such tables as those of Harting or, alternatively, from approximate formulæ, as published in the "Instrumentenkunde," and that, having decided on the approximate form, the actual form is then computed by the trigonometrical reckoning through of characteristic rays? Of optical treatises in the English language that deal with the trigonometrical computation of optical systems there exists not one apart from the very brief discussion of the subject by the late Professor Silvanus P. Thompson in his appendix to Lummer's "Photographic Optics."

The trigonometrical method elaborated by Professor Ludwig von Seidel at the request of Dr. C. A. von Steinheil lays no claim to mathematical elegance. It is from the academic point of view a brute force method.

If the algebraic formulæ of the requisite order for computation purposes were sufficiently simple, there might be some justification for the large amount of attention that has been devoted to them in text-books, but the formulæ that are generally given are no more than approximate formulæ preliminary to the actual computation. A spherical aberration formula of the third order is an unwieldy affair of impracticable proportions. It is very desirable that the attention of our mathematicians should be directed upon the actual computation of optical systems until such time as they have acquired a real knowledge of the problems involved. Mr. Everitt's paper will, I hope, serve to show those who are studying the subject that the best kind of work can be done without an expert knowledge of mathematics, although such a knowledge is desirable in the case of research work.

MR. H. DENNIS TAYLOR writes :—

With regard to the fulfilment of the so-called "sine condition" when the objective is free from coma in its lateral or oblique images, I find the following words in Mr. Everitt's paper: "Since the magnification varies as the focal length, the sine condition expressed in its simplest form is that the magnification due to the lens system shall be constant over the whole aperture." This idea, that if a lens shows coma it means that its different zones have different focal lengths, was also emphasised by the late Prof. Silvanus Thompson ; but according to my own view of it I regard this way of interpretation rather overdone and far-fetched, for the following reasons :

In the first place, it assumes that each concentric zone of the lens has its own focus, which can be located. The fact is (assuming astigmatism not to complicate matters by its presence) that each zone paints a more or less large *circle* of light, and while the rays passing through the particular zone at two points *a* and *a* (see Fig. *a*) cut by the primary plane focus together at a point A in the comatic circle, two rays refracted through the zone at *b* and *b* in the secondary plane focus together at the point B in the comatic circle, so that if C be the point where a proper sharp image *should be* formed, then A is 3 times as distant from C as B is; also the two rays from the two points *d* focus at D and the two rays *e* at E. The rays distributed round one circumference of the lens zone are distributed *twice* round the comatic circle. In what sense, then, can the zone in question be said to possess a focus at all? Is it the centre of the comatic circle?

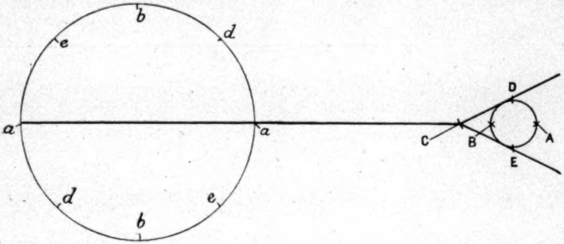

Fig. *a*.

If so, it is a vague abstraction, for this comatic circle cannot be made to focus at all. Should the plane on which this comatic circle is projected be moved further away from, or nearer to, the lens, then its double nature shows up as this coil of Fig. *b*.

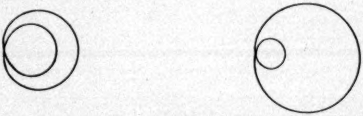

Fig. *b*.

Altogether I fail to see why the phenomenon of coma should not be regarded as a manifestation on its own basis, from the point of view of definition a fault more serious than any other optical defect, inasmuch as it prevents sharp definition of all points or lines in the image, whereas astigmatism at least allows lines running in one certain direction to be defined.

Finally, while we must all feel grateful to Mr. Everitt for his valuable paper, I feel I must call in question his apparent tendency to refer every question to German works on optical science, just as though nothing had been done in this country to elucidate such questions, and I trust that in future years such anti-British leanings will be still less justified than they can possibly be held to be now.

Sir Howard Grubb writes :—

I am greatly obliged for the opportunity of perusing the advance copy of Mr. Everitt's most valuable paper. It has not been possible for me to closely follow out all his arguments, but he has certainly succeeded in condensing in quite a short paper a large amount of information which cannot fail to be a great help to the practical lens maker.

I must confess to a considerable amount of diffidence in offering criticisms on any part of his paper, as it deals more especially with the design and construction of the smaller classes of objectives, of which my experience up to the present is small. My lifework has been spent mostly on astronomical objectives of the larger sizes, in which, though the principles involved may be the same, the *modus operandi* is very different. I may be perhaps permitted to make a few remarks, though these will apply mostly to large objectives.

As regards the cementing of object glasses, I see Mr. Everitt recommends that none over 3 ins. diameter be cemented. I think he might fairly well extend this to 5 ins. in certain cases. For instance, in the case of some forms of periscopes, where four or more objectives are necessary, we find great advantage in cementing the object glasses though some of them are 5 ins. in diameter. The saving of light thus effected is quite appreciable. In larger sizes still we have successfully used the cementing, even up to 15 ins. diameter. In these cases, however, we do not use Canada balsam or any cement that hardens in time—we use sterilised castor oil. One example is that of the 15-in. object glass used by the late Sir W. Huggins, F.R.S., which, I think, has been between 40 and 50 years cemented. If this castor oil is used, it should be brought to the temperature of melting tin (easily ascertained by throwing in a small piece of tin foil) in order to sterilise it. Curiously enough this device was used by my father long before Pasteur announced his principles of sterilisation.

As regards the construction of large mirrors and objectives, I had in some cases help from distinguished scientists conversant with the higher optical mathematics, but the finished object glass was very seldom completed to the exact curves as computed by the mathematician.

When we consider the high degree of accuracy of surfaces required it appears extraordinary that it should be possible to produce surfaces with the necessary perfection.

Suppose we take the case of a 24-in. object glass, of say 30-ft. focus, and consider a ray which falls on a zone that is, say, half-way between centre and edge (6 in. from centre). If there be any surface error which causes that ray to intersect the axis at 1/10 in. greater or less distance than rays from other zones, this error ought to be detectable, and yet such an error will exist if the deviation on any part of that zone is equal to as much as a second of arc, and for smaller zones a smaller error will cause the same difference in focus.

The final work on large object glasses may be divided into two distinct parts, (1) the detection and localisation of the final errors, and (2) the removal of same. (1) can be undertaken by anyone of good average intelligence and scientific training. It can be communicated from one individual to another, and various suggestions of a most valuable nature have been from time to time put in print on this subject, more especially by American workers, but (2) I have always looked upon as more a work of art than a mechanical operation.

Just as a person might watch an artist painting a picture for weeks and months, so might anyone watch the progress of the figuring of large mirrors or object glasses and yet be quite unable to do the work themselves. One of the most famous of our art critics told me himself that, while he possessed an instinct that enabled him to judge of the merits or origin of a picture, he was utterly unable (though he tried persistently) to paint a decent picture. So it is with the final figuring of lenses and mirrors. A scientist or skilled observer who can best detect and point out faults in an object glass would probably be quite unable to remedy those faults even though he be conversant with the *mode of operation*. I think Alvan Clarke, the great American maker of large object glasses, must have had a similar idea, for a friend of mine who once watched him figuring a surface remarked to him, "Why, if I worked as you are doing, I could polish a great hollow in the surface." Clarke replied, "Yes, no doubt; *but I won't.*"

My experience goes to show that it is not everyone who is possessed of the instinct or art necessary for the perfecting of large optical surfaces, and that no amount of education or instruction will enable one who is not possessed with this particular gift to deal successfully with this work.

I should like to add my testimony to the usefulness of what is called in the paper the Hartmann system of testing, more particularly for object glasses corrected for photographic rays. We have used what is practically this system for many years at our Works, but we employ for the purpose a set of holes in one straight line across the face of the object glass instead of the sets of concentric holes described in the paper; assuming that the surface is one of revolution, we find it sufficient for our purposes.

DR. R. A. SAMPSON writes :—

One of the greatest difficulties in optical matters is for the mathematician and the practical man to get into effective touch. The mathematical discussions are nearly always too complicated to be of much practical service, and on the other side the mathematician finds it difficult to gather what may be reasonably disregarded for ordinary purposes. If he turns to a work like Steinheil and Voit's "Handbuch," its very completeness increases his difficulty. From this point of view one welcomes a paper like Mr. Everitt's. Mr. Everitt is at the same time a practical man and a good mathematician, and though professedly his paper says nothing new which cannot be drawn from well-known text-books, it sets out the most relevant considerations very clearly. As a point of criticism, I should say, however, that I think astigmatism and curvature of the field are dismissed too lightly. It is quite true, as Mr. Everitt says, that with an ordinary doublet almost nothing can be done with them. The radius of curvature of the field is 2/5 of the focal length, and the separation of the focal lines at different points is a fixed amount.

Still, the matter is only secondary in importance to spherical aberration and coma in the increasingly important class of astronomical photographic lenses. The effect of the error here is to scatter the light over an irregular area, besides spoiling definition, and by this the images of the faintest stars are lost. Indeed, from experience with the "Astrographic Catalogue," the best practice sets deliberately *within* the focus for axial rays, so that the light above the axis is somewhat diffused, and the best definition is about 40′ from the axis. Without this precaution the rapid decline of efficiency from the centre of the field outwards would be intolerable. Now astronomical lenses have been made that work well up to 8° or 9° from the axis. It is certain that the demand will be made for flatness of field and circular images of extra-axial points in the future, even though this can only be secured by separating the constituent lenses of the object glass.

In describing the appearance of images found by imperfectly corrected lenses, I think reference ought always to be made to the effect of diffraction. Without this, *e.g.* uniformity of illumination is a fallacious guide. The whole of these effects are beautifully illustrated by photographs in Messrs. T. Cooke and Sons' well-known brochure on the testing of lenses.

I think it is worth while to recall that the nodal points were not introduced by Gauss ; they were added by Listing. Listing was a physiologist, and it was his interest in the eye as an optical instrument, in which the initial and final media are different, that led him to introduce them. But this is a small point of criticism. I have read with pleasure the whole paper, which is unavoidably compressed, and upon the whole very lucid and useful.

Lt.-Col. J. W. Gifford said, in regard to residual aberrations : The author tells us under (1) that the axial pencil will be free from chromatic aberration for the two wave lengths C and F, and under (2) that the axial pencil will be free from chromatic difference. Has he tried using the edge ray for the achromatism, or better still, a 2/3 edge ray ? At such a height from the vertex the correction is representative of a far greater area. But the achromatism has to be approached in the first instance from the axis, and, as far as I can see, Mr. Everitt has not shown us how to approach it. Indeed, he tells us, under the heading "Actual Calculation of the Objective," that he falls back on approximate radii and the use of tables. The following formula will remind everyone of an old and tried friend, viz., $\frac{1}{f} + \frac{1}{f'} = \frac{1}{\mathrm{F}}$. You will see presently its significance when used in conjunction with the axial trigonometrical trace as given by Steinheil and Voit. By our old formula, $f = 1 - \frac{(\mu' - 1)\,\delta\mu}{(\mu - 1)\,\delta\mu'}$, $f' = 1 - \frac{(\mu - 1)\,\delta\mu'}{(\mu' - 1)\,\delta\mu}$, when $\mathrm{F} = 1$. Now, for the purpose of the axial trace, let H, the height from the vertex at the surface of incidence, be such that for parallel incident ray the angle made with the axis by the emergent ray of the combination be equal to one second of arc, then the value of $\frac{1}{f}$ in seconds of arc will be the angle made under like conditions with the axis by the first element of the doublet ; $\frac{1}{f'}$ that by the second element. Now if, having chosen our F, we make H = F, then we have the angle made with the axis on emergence (Mr. Everitt's λ_2) $= \frac{\mathrm{H}}{\mathrm{F}} = 1$, that is one second of arc. The error introduced by such a makeshift is negligibly small, and is due to the difference between the sine and arc of one second. Thus we start the trace :—

Data.

R_0 is arbitrary. $\beta_{0,2} = \frac{1}{f}$ $\delta_2 = \beta_{0,2} - \delta_0.$ $\mathrm{R}_2 = \frac{(\mathrm{H}_0 - t_1\,\delta_0)\,\mu - 1}{-\,\delta_2 + \delta_0\,(\mu - 1)}.$

$\beta_{4,6} = \frac{1}{f'}.$ $\delta_6 = \beta_{4,6} - \delta_4.$ $\theta_6 = \frac{\delta_6}{1 - \mu'}.$ $\mathrm{R}_6 = \frac{(\mathrm{A}_4' - t_5)\,\beta_{0,4}}{\theta_6 + \beta_{0,4}}.$

$\theta_0 = \frac{\mathrm{H}_0}{\mathrm{R}_0}.$

$\phi_0 = \frac{\theta_0}{\mu}.$

$\delta_0 = \theta_0 - \phi_0 = \beta_{0,0}.$

I must apologise to Mr. Everitt for using Steinheil and Voit's terminology with my own abbreviations—I am rather lost in any other. He appears to have used von Seidel's. Since von Seidel's time, λ has become conventional for a wave length. Then again

ϕ is less likely to be confused with θ than ϕ with ψ. Again, in England we generally spell thickness with a "t," not with a "D" (Dicke). The trace I have given is for a cemented doublet. At this point let me revert to the choice of lines in the spectrum for which focal lengths are to be equalised for the colour correction. Mr. Everitt takes C and F. Has he tried D and F (Herschel) or, better still, D and E?

In the testing of objectives I would prefer, where artificial light is necessary, to use a Nernst lamp and pinhole, which gives the whole visible spectrum, instead of the Cadmium spark. If objectives are correctly constructed, for small objectives up to 3 inches of aperture figuring is unnecessary in my experience.

Mr. Everitt refers to von Seidel, also Steinheil and Voit, and Gleichen. von Seidel first adopted the system of trigonometrical trace in 1866, and the "Handbuch der Angewandten Optik" appeared in 1891. In 1900-1 Mr. E. M. Nelson devised a system of trigonometrical trace quite independently, and sent me a copy which is still in my hands. I made use of it a great deal at one time. Finally, a paper was read by me before the Royal Astronomical Society, and appeared in the "Monthly Notices" for December, 1908, which gave the algebraic first approximation and the complete trigonometrical trace for a triplet of three different kinds of glass. When recourse must be had to tables, and the glasses are comprised in Messrs. Chance's list, the tables of Messrs. Smith and Cheshire should be a great help. I have checked these tables by trigonometrical trace, and they are quite sufficiently correct for the purpose for which they are intended.

$$A'_o - R_o = \frac{R_o \phi_o}{\delta_o}.$$

$$\theta_2 = \frac{(A'_o - R_2 - t_1)\,\delta_o}{R_2}.$$

$$\phi_2 = \theta_2 \mu.$$

$$\delta_2 = \theta_2 - \phi_2.$$

$$\beta_{o,2} = \delta_2 + \delta_o.$$

$$A'_2 - R_2 = \frac{R_2 \phi_2}{\beta_{o,2}}.$$

$${}^* F_{o,2} = \frac{H_o}{\beta_{o,2}}.$$

$$\phi_4 = \frac{\phi_2}{\mu'}.$$

$$\delta_4 = \phi_2 - \phi_4.$$

$$\beta_{o,4} = \delta_4 + \beta_{o,2}.$$

$$A'_4 - R_4 = \frac{R_4 \phi_4}{\beta_{o,4}}.$$

$$\theta_6 = \frac{(A'_4 - R_6 - t_5)\,\beta_{o,4}}{R_6}.$$

$$\phi_6 = \theta_6 \mu'.$$

$$\delta_6 = \theta_6 - \phi_6.$$

$$\beta_{o,6} = \delta_6 + \beta_{o,4}.$$

$$A'_6 - R_6 = \frac{R_6 \phi_6}{\beta_{o,6}}.$$

$$F_{o,6} = \frac{H_o}{\beta_{o,6}}.$$

MR. CONRAD BECK said he was sorry he had not heard or read the original paper, and therefore any remarks he might make would merely have reference to the observations which he had heard that evening. He had been extremely interested in Mr. Hasselkus's description of the object glasses of which he had put particulars on the blackboard, because it brought into prominence the fact that the telescope object glass was not an item by itself, but must be made in connection with the eyepiece with which it was to be used, and further, must correct the defects of the eye itself. Of course that was one of the things which made it so difficult to make, and the preparation of the details of curves and refractive indices so as to make a simple series of tables for the manufacture of object glasses was apt to be misleading unless it was known also what eyepiece was to be used. Some of the corrections obtained in the use of large aperture object glasses were due to opposing the errors in an eyepiece with those in the object glass, and it must follow that, in order to produce instruments with these large angles of view and large apertures, it was not possible to rely too much on perfectly corrected separate systems in order to produce the finest results. In regard to Mr. Chalmers' remarks, the criticism he made was that these formulæ obtained from the two first terms of the expansion of the sine, while useful for systems of small aperture and small angle of view, were often misleading for systems not thus limited—such formulæ appeared to be a ready means of giving the necessary shape for an object glass. But they did not carry one very far, because, unless one took a cemented doublet, the equation was an indeterminate cubic instead of a readily solved quadratic equation, and a number of shapes would be found directly the cemented doublet was departed from. The introduction of a further condition might be employed to turn the cubic equation into a quadratic, but the choice of such a condition became a matter of great skill. When this was done the equation method was not accurate enough except for small apertures. The study of this method of equating the errors to zero or to a fixed quantity was, however, the best educational method, as it gave the student a grasp of the principles of correction that could not be obtained by trigonometrically plotting rays, and it would be desirable if some better equating method could be discovered. Naturally some of the other corrections had not been touched upon that evening—for instance, the variation in the spherical aberration of different colours, which, in some of the large-aperture object glasses, became quite an important matter. The whole question of telescopes had changed so much since the days when small-aperture lenses as used for astronomical telescopes with an aperture of perhaps $\frac{1}{12}$ to $\frac{1}{20}$ focus were in use, while at the present day object glasses ran up to $f/4$, and complicated the whole question.

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* The last step of trace for single convex and unnecessary for the doublet. I notice that Mr. Everitt in the trace for edge ray uses the sine at this stage for $\beta_{o,2}$; is not the tangent more correct?

Mr. A. E. Conrady writes :—

Mr. Everitt's paper seems to me deserving of a hearty welcome, because it renders accessible to members of the Society the rigorous method of trigonometrical calculation which must be the foundation of sound computing work.

The paper is, practically, an abbreviated version of Steinheil and Voit's method, with the omission of the formulæ applicable to the general case of rays not cutting the optical axis.

The weakest point of Steinheil's method is to be found in his treatment of achromatism, which he establishes for the paraxial zone only, treating the whole spherical difference as " offence against the Gauss condition." This is all the more strange because one of the Steinheils, some thirty years before the publication of the book, correctly stated in a lecture that, in order to reduce the chromatic aberration to a minimum, the correction should be carried out for the zone corresponding to 2/3 of the full aperture. It is to be regretted that Mr. Everitt followed Steinheil in this respect.

A graver objection to Steinheil's book, as well as to the present paper, arises, however, from the fact that the influence of the eyepiece on the desirable correction of the object glass is totally ignored, being apparently assumed to be negligible. This is very far indeed from being the case, as all eyepieces capable of giving a large field of view are more or less under-corrected, both spherically and chromatically. The spherical defect *can* be corrected by " figuring," but the chromatic residuals must be provided for in the proportioning of the object glass or in the selection of glass of suitable dispersion. Taking the latter method first, we will assume that an object glass computed to be perfectly corrected in itself is to be adapted by change of the dispersion of the crown glass to give perfect achromatism at magnification 20 with various usual eyepiece types. Assuming that the original crown glass had a dispersion between C and F of ·00860, then this dispersion would have to be reduced to ·00846 to adapt the object glass for a × 20 Ramsden eyepiece, to ·00831 to adapt it to a × 20 Huyghenian eyepiece, and to ·00796 to suit a good type of 4-lens terrestrial eyepiece of × 20. In addition, a considerable amount of " figuring " would have to be done to remove the spherical undercorrection.

The proper method is, however, to compute the telescope *as a whole*. Making such a computation for an object glass of 20″ equivalent focal length and 2″ aperture, made from Hard Crown and Dense Flint, the following approximate data result :

A. Objective corrected in itself.		Objective overcorrected to compensate × 20 terrestrial eyepiece.	
Biconvex Crown	$r_1 = +314$ mm.	Biconvex Crown	$r_1 = +314$ mm.
8 mm. thick	$r_2 = -157$ mm.	8 mm. thick	$r_2 = -135$ mm.
Flint Meniscus	$r_3 = -160$ mm.	Flint Meniscus	$r_3 = -138$ mm.
5 mm. thick	$r_4 = -670$ mm.	5 mm. thick	$r_4 = -615$ mm.

It is obvious at a glance that the alteration required is not a small matter, even when the magnification is so high as 20 ; in the more common case of terrestrial magnifications of 10, 5, or even 3, the influence of the eyepiece grows in inverse proportion, until at the last-mentioned power it may be easier to obtain a good result at minimum cost by using an unachromatic object glass and a compound erecting lens than *vice versa.*

It may be stated, in conclusion, that it may even be advisable sometimes deliberately to introduce departure of the object glass from fulfilment of the sine condition, in order to compensate coma effects of the eyepiece, which are very apt to accompany the artificial flattening of its field.

THE PRESIDENT said that, before calling upon Mr. Everitt to reply, he thought he must say that the Society was to be congratulated upon the valuable contribution which it would now be able to make to the literature of Technical Optics. In connection with the subject of designing, he had just communicated to the "Journal of the Royal Microscopical Society" two very interesting letters written by Professor Abbe, about 1886, to John Ware Stephenson. The latter was an English microscopist who had suggested to Professor Abbe that the principle of homogeneous immersion should be tried. He believed that the principal idea dominating Stephenson's mind when he made the suggestion was the possibility of getting rid of the correction collar, but he had stated clearly at the same time that it would enable an experiment to be made for the purpose of determining whether it was not possible to increase the resolving power of the microscope by using this principle of homogeneous immersion. Those letters were also very interesting from the point of view of optical design, because, as they knew, the homogeneous immersion objective was one of the most complicated optical systems in existence; it contained, he believed, something like nine or ten lens elements, and took up a cone of rays of the order of 160°, so that it was a very difficult problem for the computer. Abbe's letters gave convincing evidence that the system was completely worked out upon paper before it was sent to the workshop; and, what was more, when it had been made in the workshop and tested, it was found that the mathematical results which had been arrived at were completely realised in the practical construction. That showed that, even so early as that, Professor Abbe had succeeded in introducing a much higher school of design than opticians generally had used—with perhaps the single exception of Steinheil, about whom possibly Mr. Everitt would tell them something in the course of his reply. Prior to Abbe's time the workman had generally been relied upon to test the work of the designer, but Abbe substituted a more efficient test—that of the mathematician. Now that was an exceedingly important difference, and one which must be realised to the full to-day if the English optical industry was going to raise its head and once more occupy its former proud position. That could only be done by adopting the best and shortest methods of arriving at results in every case. That meant that, in the question of design, one must not be content with second-class designing—that was to say, with a school of design which was not quite sure of its results until a workshop test had been made. They must have a school of design which was able to work out upon paper the system desired, and that must be tested by mathematical computation rather than by the operations of the workman. That was an important point, and one which must be driven home. They must have a school of designers who would do their work so well and so reliably that when they had once got out their results the manufacturer would feel justified in putting the tools down and setting in hand the necessary operations for producing the instruments in quantity, without waiting for a sample to be made and tested.

Reference had been made from time to time in the course of the discussion to the tables of Harting. He did not think sufficient prominence had been given to the fact that some time ago Messrs. Willmott and Whitwell had brought those tables up to date for those glasses of Messrs. Chance which could be obtained at the present time. Those tables had been got out with a great deal of care, and the results had been published in the pages of *The Optician*. He was sorry that they had not been more widely used. He would like to see those tables printed on a card, and hung up in every workshop.

Sir Howard Grubb had written a great deal upon the subject they had been discussing, and he desired to quote a passage from a paper read by him before the Royal Institution in 1886, on " Telescope Objectives and Mirrors." That passage was very significant and important, and it was as follows :—" Some years ago the Royal Society commissioned one of its members to draw up tables for the use of opticians, giving the curves required to satisfy the conditions of both corrections for all refractive and dispersive indices. A considerable amount of labour was expended on this work, but in the end it was abandoned, for it was found that the calculation of these curves was founded on the supposition that all surfaces produced by the opticians were truly spherical ; while the fact is, a truly spherical curve is the exception, not the rule. The slightest variation in the form or figure of a curve will produce an enormous variation in the correction for spherical aberration, and it was soon apparent that the final correction for spherical aberration must be left to the optician and not to the mathematician. *Object glasses cannot be made on paper.* When I tell you that a sensible difference in correction for spherical aberration can be made by half an hour's polishing, corresponding probably to a difference in the first place of decimals in radii of the curves, you will see that it is practically not necessary to enter upon any calculation for spherical aberration. We know about what form gives approximate correction ; we adhere nearly to that, and the rest is done by figuring of the surface."

Sir Howard Grubb was speaking of large telescope object glasses. For practically all other optical constructions the optician is severely limited to the use of spherical surfaces.

AUTHOR'S REPLY.

Mr. P. F. Everitt said that he felt rather overwhelmed by the weight, the authority and the kindness of the criticisms to which he had listened.

Mr. Hasselkus had referred to the case of a binocular objective system and had had the kindness to let him glance through his notes before speaking. In the first place, Mr. Hasselkus had started with a particular eyepiece constructed of glasses which did not appear to be those most suited to the construction. This eyepiece had certain aberrations which the objective was required to compensate as well as to perform its own special functions. Naturally the example worked out in the paper was not designed to compensate the aberrations of this or any other eyepiece, and consequently the comparison drawn between it and another objective put forward by Mr. Hasselkus was not to its advantage ; further, he did not think that any optical designer would consider it fair to draw such a comparison.

Sir Howard Grubb had assumed that the methods described were intended to apply to small objectives only ; they had been used successfully on some of the largest objectives constructed in Europe, with the modification that the aberrations found on the first testing had been used to derive a corrected set of curves to which the objective was altered. This modification was necessary owing to the small but continuous variation of refractive index from centre to edge, found in all really large discs of glass.

Sir Howard Grubb's remarks on cementing had been most valuable ; of course the limit of the size of objective it was advantageous to cement depended upon the standard of optical perfection aimed at to a large extent, and the method of using sterilised castor oil considerably reduced the mechanical strain to which the cemented lenses were subjected.

Dr. Sampson considers that the problems relating to astigmatism and curvature of field should be accorded more consideration. I agree that, when dealing with astrophotographic objectives, they should receive as much consideration as coma, and regret that, in dealing with such a large subject as the design of telescope objectives, it was unfortunately not possible to consider any special types within the limits of a paper to be read before this Society.

It cannot be too widely known that Dr. Sampson† has devised a most ingenious method of computing the aberrations of optical systems which is peculiarly adapted to the problem of astrophotographic objectives.

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† A New Treatment of Optical Aberrations, by R. A. Sampson, F.R.S. Phil. Trans. Ser. A., Vol. CCXII, pp. 149-185.

Lt.-Col. Gifford rightly demands a rigorous correction of the chromatic aberration of a ray incident at 2/3 the distance from centre to edge of the objective. The same result is obtained by making the axial and edge aberrations equal and of opposite sign (balancing the aberration) without the labour of computing an additional pair of rays.

With regard to the choice of colours for which the achromatism is to be carried out, I consider that C and F are preferable for objectives used for general purposes, and believe that such is the general practice of the majority of optical designers.

Lt.-Col. Gifford does not appear to have quite grasped my purpose when using the cadmium spark for testing chromatic corrections. For visual observation with a direct vision prism the spark is not essential; but this test is nothing more than a preliminary examination. The real test is charting the colour curve by means of the Hartmann method of extra-focal measurements, and here it is essential that the wave-lengths of the colours examined should be known, and for this purpose the cadmium spark is extremely good, giving, as it does, a nicely spaced series of bright lines ranging from red to violet.

Mr. Conrady's objection to the Steinheil and Voit "Handbuch," that the variation of the chromatic aberration with aperture is disregarded, may be met by balancing the axial and edge aberrations, which does not involve computing a pair of rays for 2/3 of the aperture.

Mr. Conrady (like Mr. Hasselkus) also demands that the objective shall compensate the aberrations of a badly designed or inferior type of eyepiece. This practice of saddling the poor objective with the task of annulling the consequences of the adoption of an inferior type of eyepiece can only be carried out for one particular eyepiece, and the correction so obtained only holds for one particular magnifying power. As eyepieces have been constructed for many years which are far superior to the ordinary Ramsden and Huyghenian types to which Mr. Conrady refers in his numerical example, I consider it to be a very poor policy not to make use of them. If the eyepieces are properly corrected, the magnification of the telescope can be changed without sacrificing the state of correction that has been obtained. I feel sure that Mr. Conrady will agree with me that these better types of eyepieces should be made use of to a far larger extent than is now done. In special instruments of one magnifying power only, in which the demands on field and aperture are pushed to the utmost, it is frequently advisable to compute the instrument as a whole, and then it is no longer strictly correct to speak of objective and eyepiece except as a matter of custom. Such special cases could hardly be included within the limits of a single paper.

Mr. T. Smith has given a very valuable contribution setting forth an alternative method of design. In the absence of any past experience giving a preliminary set of curves which it is known will be fairly close to the desired result, Mr. Smith's method will usually be found preferable. It must, as he says, be used intelligently and with due regard to its limitations; the values so obtained should always be checked by trigonometrical methods whenever the relative aperture is large, deep curves involved, or there is only a small difference in refractive index between two successive media. Mr. Smith's aberrations do not agree with mine, but this is to be expected from a solution differing to quite a considerable extent in the actual radii obtained; incidentally, his error against the sine condition is appreciably larger, but not seriously so.

These small differences between solutions are in many cases merely of academic interest and are not of practical importance.

The other contribution, by Mr. S. D. Chalmers, does not differ so widely from the methods given in the paper. There are a few minor defects which should be pointed out, and which Mr. Chalmers may remedy at a later date.

Mr. Chalmers has corrected chromatic aberration by an alteration of the last radius, and in doing so unavoidably upsets the focal length of the objective; this involves enlarging or reducing the scale of the system, with possible trouble in actual construction from considerations of substance. Further, I think a skilled computer can correct a system more rapidly, using the method given in the appendix for obtaining chromatic correction, and altering the curves from past experience to obtain spherical correction, than by following Mr. Chalmers' methods, although in less skilled hands the reverse may be expected.

In conclusion, I wish to express my special thanks both to Mr. Smith and Mr. Chalmers for permitting such valuable contributions to appear with this paper.

The Eye in Relation to Practical Lighting Problems.

By J. S. DOW.

Mr. J. S. Dow gave a brief lecture on "The Eye in Relation to Practical Lighting Problems." Referring to early work on physiological optics, he said that many of the conclusions reached had an intimate bearing on modern lighting problems, but the information was not as a rule conveyed in a form which could easily be applied. For example, numerical data, giving actual figures for the order of brightness at which various effects occurred, were scanty.

Mr. Dow mentioned some general considerations guiding one in the choice of lighting arrangements. The eye had been developed to daylight conditions, and the brow and upper lashes shielded it from light from above; similarly, very strong light entering from below might be inconvenient, and snow blindness was sometimes ascribed to this cause, as well as to the predominance of ultra-violet light in mountainous regions. People sometimes compared indirect lighting to daylight; but so far as variations in brightness were concerned, the divergences met with in natural light were far greater than those occurring with artificial lighting, and the illumination derived from indirect lighting was exceptionally uniform. Exposure to the intrinsic brilliancy of the sun, or to direct reflection of its rays from polished objects (such as the surface of the sea) might be prejudicial to the eyes, and even the full sunlight illumination of a white surface, giving an equivalent brightness of about 10,000 ft.-candles, appeared to be beyond the glare limit, at least for people living in northern latitudes.

He had recently been trying to calculate the value of moonlight from the observed value of full sunlight, and had found that the results, assuming the distance of the sun from the earth and moon, agreed quite closely with the value of full moonlight actually observed — namely, about 1/50 ft.-candles. The eye thus experienced in nature illuminations varying in the ratio of 1 : 500,000, and was marvellously equipped for adaptation to these extremes, provided the change occurred slowly and time was afforded for the process of adaptation to take place.

It was becoming increasingly recognised that the chief consideration in artificial illumination, with a view to avoiding glare effects, was to eliminate sudden transitions from brightness to darkness. It was at one time suggested that all sources should be so screened that their absolute brightness did not exceed 2.5 candles per square inch—about the value of the normal white sky. But this was not enough. The order of brightness causing glare depended on the state of adaptation of the eye; even a match, struck in a dark lane near the eye, produced temporary dazzle, and in the present darkened condition of the streets the inconvenience caused by glare from motor-car headlights or unshielded powerful lamps had been made manifest.

The whole question had been recently investigated experimentally by Mr. P. G. Nutting in the United States. He gave as the absolute glare limit a brightness about four times that of white paper illuminated by full sunlight; such a brightness is glaring whatever the contrast. The ratio of brightness which the eye can tolerate varies somewhat according to the illumination, but in general should not exceed 100 :1. This is a useful rule for practice. When strong local lighting is employed there should be moderate general illumination to prevent the ratio between the brightness of the strongly-lighted work and the relatively subdued surroundings exceeding the 100 : 1 ratio.

The progressive tendency towards higher illuminations during recent years made it important to determine what illumination was needed for different classes of work. Mr. Dow gave some instances of this increase, *e.g.*, in street-lighting, and on railways, where the order of illumination prevailing about 50 years ago, or even earlier, could be estimated and compared with that ruling to-day. Nutting's results were useful as showing how capacity to distinguish light and shade increased with rising illumination up to a certain point, but eventually fell owing to glare. These data supported the view that for general purposes, when one had only to distinguish the outlines of large objects and black and white contrasts, a relatively low illumination would answer. Thus, in passages, yards, &c., ¼ of a foot-candle was often sufficient; for reading good black type on white paper a minimum of 1-2 ft.-candles satisfied most people's requirements; but for observing very fine detail and small contrasts, *e.g.*, in sewing and knitting, watchmaking, engraving and the like, as much as 10-50 ft.-candles might be wanted. It was important to observe that when the work was dark in hue the illumination should be correspondingly increased in order to attain the requisite brightness. In order that a moderate illumination might be successfully used, the elimination of glare and severe contrast was very important. In this connection Mr. Dow referred to some experiences in the lighting of cinema-theatres and rifle-ranges and local industrial lighting where it had been found that a severe contrast between the brightness of the object viewed and the surroundings, interfered with acute vision. It was now being suggested that in cinema-theatres, and in connection with the stage, the auditorium preferably should not be left in complete darkness, so as to keep within the 100 : 1 ratio.

Mr. Dow pointed out that for many kinds of work shadows were essential in order that the object might stand out in relief; this applied especially to textile operations, and in some kinds of printing; in the case of carving and embossed work, shadow-conditions were specially important. Nutting had suggested that in general the best conditions were secured by a mixture of 30 per cent. diffused and 70 per cent. direct light, such as was commonly met with in daylight. But there were some special cases in which practically uni-directional light was wanted.

In conclusion, Mr. Dow showed a series of lantern slides illustrating, by photographs of actual lighting installations, many of the points dealt with in his lecture.

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